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Discussion of “Copulas: Tales and facts”, by Thomas Mikosch

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The essence of the provocative statements in Mikosch (2005) is that copulas do not add much to our understanding of multivariate probabilistic questions. When first presented at the 4th International Conference on Extreme Value Analysis in Gothenburg, 15–19 August 2005, most discussants agreed with this conclusion. We do not necessarily disagree, but in the interest of the intellectual debate, we nevertheless want to advance a few arguments in defense of the copula concept. Copulas are a way to separate the dependence structure from the marginal probabilities. In our discussion, we first give two examples from economics as to how this separation can be useful. Subsequently, we argue that if one tries to model the dependence structure, the use of a specific copula should be motivated by the problem at hand.

1 When to use copulas?

In studying multivariate statistical problems it is sometimes useful to separate the dependence structure from the marginals, in order to focus on only one of the two. We give an example of either case.

We start with an example of genuine interest in copulas in the banking industry. Banks are more heavily supervised than other parts of the economy. The reason is that banks provide an important positive (unpriced) externality to the macro economy by their maintenance of the payment and settlement system, for which the systematic stability is crucial. The Basle II accord provides the framework within which the supervision takes place to ensure

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the systematic stability. Copulas offer a practical concept by which one can characterize the systematic stability of a particular banking system, whereas the marginal risks are captured by the VaR (Value at Risk)[1] concept. Having a measure of systemic risk separate from the marginal risks, even if alternative measures are available, is important for comparison of banking systems and supervisory activity levels in different countries. Given the deficiencies of the correlation concept and the apparent non-normality of the marginal risks, the copula concept offers an attractive alternative. The problem lies in choosing the right copula.

Next, we show how the copula concept can be useful to characterize the marginal distribution in the well known Colonel Blotto Game. The game was originally introduced and solved by Borel (1921). The Colonel Blotto Game is a one-shot game in which players, A and B, compete by allocating simultaneously forces across a finite number n, $n \geq 3$, of homogeneous battlefields. The force allocated to each battlefield must be nonnegative and each player i has a budget constraint (total number of forces). Each battlefield is won by the player who provides the larger force on that battlefield, and players' payoffs equal the proportion of battles won.

It is well known that there is no pure strategy Nash equilibrium for this game. A mixed strategy for player i is an n-variate distribution function F_i, with one-dimensional marginal distributions $\{F_i^j\}_{j\in\{1,\cdots,n\}}$, which describes the random allocation across battlefields. Borel gave a mixed strategy Nash equilibrium solution to this game. Until recently, it was not known whether the marginal distributions of Borel's equilibrium were also unique. Roberson (2006) solved this problem by isolating the marginal distribution via the copula decomposition. Then he was able to show that the marginal distributions proposed by Borel are unique.

2 How to use copulas?

Given an interest in the dependence separately from the marginal distributions, one may wonder what kind of copulas are natural in, say, economics and finance? To provide an example, take the case of two portfolios which are different combinations of the same independent risk factors, which is standard fare in financial economics. Since portfolios are naturally linear combinations of different assets, the copula should reflect this linear structure, see de Vries (2005). The affine structure puts restrictions on the relevant class of copulas. Thus, in empirical work, one is not free to choose just any copula. Similarly, economic arguments may imply that the copula should exhibit asymptotic dependence or asymptotic independence, which again restrict the classes of relevant copulas. For this reason a non-parametric approach to the dependence structure may be preferable over any specific parametric copula choice.

[1] VaR is a high loss quantile at a given low probability level.

The converse may also be the case, however. Choosing a parametric copula may be the only way to address the dependence structure in infinite dimensional problems, see de Haan and Pereira (2003). The problem with a non-parametric approach in the infinite dimensional case is that the copula is a unknown (max-stable) process, which can not be pinned down through a finite number of points. Thus one is naturally lead to specifying a stochastic process of which the parameters are to be estimated.

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