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Systemic risk and diversification across European banks and insurers

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\textbf{Abstract}

The mutual and cross company exposures to fat-tail distributed risks determine the potential impact of a financial crisis on banks and insurers. We examine the systemic interdependencies within and across the European banking and insurance sectors during times of stress by means of extreme value analysis. While insurers exhibit a slightly higher interdependency in comparison with banks, the interdependency across the two sectors turns out to be considerably lower. This suggests that downside risk can be lowered through financial conglomeration.

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1. Introduction

This paper investigates the systemic interdependencies within and across the banking and insurance sectors in times of stress. Banks and insurers are both exposed to fat-tail distributed shocks through their assets and liabilities that create linkages and common exposures. Financial innovation improves the ways in which risks can be spread and transferred from the banking sector to the insurance sector and vice versa. Diversification lowers the risk of isolated shocks for a financial entity, but may simultaneously increase the systemic risk. The credit crisis shows how problems in one part of the banking sector can easily spread to other parts of the banking sector due to these risk transfers. At the start of the credit crisis, EU banks had exposure to US sub-prime mortgages of about equal size as US banks; a perfect example of international risk diversification and contagion. Other parts of the financial sector can also be easily affected. During the burst of the internet hype, banks came off lightly while insurers carried substantial losses as a result of their equity and bond exposures. The credit crisis shows that risks are moved between the banking sector and the insurance industry by means of credit risk transfers, warranting the bailout of large insurers as well as those of banks.

As we show, risk transfers between the banking and insurance books are nevertheless a useful diversification device in times of stress. This is so, because risks of banks and insurers differ, due to the differences in their business models. Banks transform liquid liabilities of depositors into illiquid assets (loans). The foremost risk drivers of these assets are the business cycle and the interest rate. A life insurance company, on the other hand, has a better match between its asset and liability maturity structure, but a major risk is the longevity risk. It can often hold assets until maturity when the time to pay has come, covering a period that extends over business cycles. Non-life insurance risk is again different. Claim risk is largely unrelated to the business cycle, while the investment risk on the premium income is. As of today, these differences and their interrelation in times of financial hardship have received little attention.

Our main research question concerns how the downside risk in the banking sector differs from the downside risk in the insurance sector and how these are related in times of crisis. To investigate these issues, we estimate the downside dependence between combinations of financials, both within a sector and across sectors. As the risk profile of both sectors is different, we find that there is scope for diversification of worst outcomes. To understand the possible differences in cross-sector risk, we develop an analytical factor model to interpret the sources of systemic risk.
Given the importance of the payment and clearing functions for the real economy, academic research into systemic risk traditionally focuses on the banking sector; see De Bandt and Hartmann (2002) for a survey. The stability of the insurance sector is therefore of a somewhat lesser public concern than the fragility of the banking sector. The systemic importance of the insurance industry is therefore more indirect by its influence on the banking sector. AIG, for example, became a systemically important institution. It was saved because it had sold credit derivatives to the banking system on an unprecedented scale. This makes the assessment of the downside risk of banks, insurers and financial conglomerates of great interest.

Traditionally, research in the area has concentrated more on the possible benefits of mergers across sectors. Early work discusses the potential benefits from the abolishment of the Glass-Steagall Act in the US for individual firms (which forbade bank holding companies to perform insurance activities); see Laderman (2000), Berger (2000), Estrella (2001) and Carow (2001). These earlier studies conclude there are gains from diversification. But a more recent US study by Stiroh and Rumble (2006) finds that the diversification benefits are more than offset by the costs of the increased exposure to new volatile activities. Moreover, Shaffer (1985) showed that diversification may benefit individual institutions, but often increases the systemic risk.

Moving to regulatory requirements, Kuritzkes et al. (2003) argue that there is scope for a reduction of 5–10% in capital requirements for a combined bank and insurance company. Although, the regulatory framework during our sample period (BCBS 2004), does not allow for cross-hedging between business lines. The different entities of a conglomerate are supervised separately according to sector specific regulation. On the one hand, due to the two pillar system, the Basel II and Solvency II regulations fall short in recognizing the potential benefits of cross-sector mergers for containing the risks in the financial system. On the other hand, the regulatory framework does not recognize explicitly the negative effects of diversification on systemic stability.

To analyze this issue we focus on the downside risk exclusively, rather than using global risk measures, like the variance. Using global risk measures such as the variance–covariance matrix is appropriate if other aspects such as upside potential also play a role (as in asset allocation questions). The downside risk-based Value at Risk (VaR) methodology is mainstream in the banking sector. In insurance, the study of ruin has traditionally put an emphasis on downside risk issues. On the industry level and the financial sector as a whole, the emphasis is on the systemic stability. Systemic risk by its very nature is concerned with the downside risk of the system.

The downside risk focus has another advantage, as it more easily enables capturing the stylized fact that the return series of financial assets are fat-tail distributed; see Jansen and de Vries (1991). The more common assumption that returns are normally distributed considerably understimates the downside risk. Hence, given the focus on downside risk, we will not start from this premise and allow for fat tails to capture the univariate risk properties. For the multivariate question of downside risk diversification benefits and systemic risk issues, the normal distribution-based correlation concept may also dramatically fail to capture the degree of dependence. For example, one can have multivariate Student-t distributed random variables that exhibit fat tails and are dependent, but which are nevertheless uncorrelated; this is impossible for normally distributed random variables. The downside risk measures that we consider are derived from Extreme Value Theory (EVT) and easily allow for the observed non-normality. Except for Gully et al. (2001), Blikker and van Lelyveld (2002) and van Lelyveld and Knot (2009), most studies focus on US data, as in De Nicollo and Kwast (2004), and assume that the returns are normally distributed. Our empirical research is focused on European data and applies extreme value theory, allowing for fat-tail risk and asymptotic dependence. In the empirical section, we measure the downside risk and systemic dependence between combinations of financials, both within a sector and across sectors. The extreme value-based techniques avoid correlation based techniques that focus primarily on the central order statistics, but rather use the extreme order statistics as in Hartmann et al. (2004).

In the remainder of this paper, we first explain the use of the downside risk measure instead of the correlation measure. Next, we provide an economic rationale for the dependence between different financial institutions to exist, even in the limit. Thereafter, we explain the methodology, give a description of the data and present the results. Finally, we summarize our findings and draw some policy conclusions.

2. Dependence

To understand the dependence between two random variables that follow a normal distribution, it suffices to have the mean, variance and correlation coefficient, as these completely characterize their joint behavior. The correlation measure itself, however, is often not a very useful statistic for financial risk analysis for a number of reasons.

As a first reason, recall that the correlation measure can be zero, while there is nonetheless dependence in the data. Consider, for example, the two portfolios \(X + Y\) and \(X - Y\), where \(X\) and \(Y\) are two asset returns. If the two assets are independently and identically distributed, then the two portfolios are uncorrelated. If \(X, Y\) are normally distributed, the two portfolios are also independent. But the two portfolios are dependent if the \(X, Y\) are fat-tail distributed, like in the case of a Student-t distribution (with degrees of freedom above 2), as the two portfolios have their largest realizations along the two diagonals. In fact, one shows that in the Student-t case for large \(s\), the conditional probability \(P(X + Y > s | X - Y > s)\) tends to 1/2, whereas under independence the conditional probability equals the unconditional probability \(P(X + Y > s)\), which tends to zero as \(s\) increases.

A second reason is the empirical observation that the return series do not follow a normal distribution. Fig. 1a displays the daily stock returns of ABN AMRO Bank and AXA since 1992 until 2003. The Fig. 1b shows randomly generated returns from a bivariate normal distribution using the estimated means, variances and correlation from the actual data. Comparing the two plots, one sees that the outliers more or less align along the diagonal as in the above portfolio example; which is a clear sign of systemic risk. Looking univariately along the axes, moreover, note that the actual returns exhibit many more outliers than the normal remakes. This is the well known fat-tail phenomenon. If the tails are so fat that the second moment is unbounded, the correlation measure is not appropriate. For the non-life insurance industry, second moment failure is considered an important issue. This is why such insurance contracts are often capped.

A third reason is that, for our purposes, we are only interested in downside dependence, while the correlation concept is a global 2

\[ \text{This is for simplicity; the argument can also be made in a CAPM setting.} \]
Recall that it is straightforward to allow for different failure levels across firms. By defining

\[ P(A > t, B > t) \]

given that at least one firm crashes, the joint failure probability is

\[ \text{expected number of failures, given that at least one firm is failing;} \]

as our measure of downside dependence.

2.1. Downside dependence

The above discussion of the correlation coefficient identifies a number of reasons for turning to an alternative measure for identifying systemic risk. This measure preferably focuses on the interdependence between downside losses and should be robust towards fat tails. Our preferred systemic risk indicator is the expected number of failures, given that at least one firm is failing; see Huang (1992). Let \( k \) be the number of firms that crash and let \( A \) and \( B \) be the stochastic loss returns of two financials. Let \( t \) be the loss level that triggers a failure. One can easily allow for different thresholds per firm, say \( s \) and \( z \), but this reduces the clarity of presentation.\(^3\) Thus, we focus on the diagonal.

With two firms, the conditional expected number of failures is

\[
E[k|k \geq 1] = \frac{1 \times P(A > t, B \leq t) + 1 \times P(A \leq t, B > t) + 2 \times P(A > t, B > t)}{1 - P(A \leq t, B \leq t)}
\]

at the common high loss level \( t \). Note that this conditional expectation can be readily extended to more than two firms. We will use the conditional expected number of failures

\[
E[k|k \geq 1] = \frac{P(A > t) + P(B > t)}{1 - P(A \leq t, B \leq t)}
\]

as our measure of downside dependence.

In a bivariate setting, the conditional failure expectation minus one equals the conditional probability on a systemic crisis. Since, given that at least one firm crashes, the joint failure probability is

\[
\frac{P(A > t, B > t)}{1 - P(A \leq t, B \leq t)} = E[k|k \geq 1] - 1.
\]

Hence, alternatively we refer to (1) as the measure of systemic risk. Hartmann et al. (2004) provide further motivation for this measure.

Unless one is willing to make further assumptions, as in the options based distance to default literature, it is impossible to pin down the exact level of \( t \) at which there will be a failure, or at which supervisors declare the institution financially unsound. For this reason, we do take limits in the theoretical analysis and consider

\[
SR(k) \equiv \lim_{t \to \infty} E[k|k \geq 1].
\]

Extreme value theory then shows that, even though the measure is evaluated in the limit, it nevertheless provides a reliable benchmark for the dependency at high but finite levels of \( t \).\(^4\) Note that for the introductory example where \( A = X + Y \) and \( B = X - Y \) and if the two factors \( X \) and \( Y \) are i.i.d. Student-\( t \) distributed, then \( SR(k) = 4/3 > 1 \). But in the case that \( X \) and \( Y \) are i.i.d. Normally distributed, \( SR(k) = 1 \). Moreover, note that the measure does not require bounded second moments and zooms in on the downside risk exposures.

Recently, the parametric approach to dependency by means of copulas has gained some popularity. In the interest of robustness we prefer not to choose a particular copula and follow the non-parametric multivariate EVT approach. Note that the connection between the two concepts in the limit is as follows

\[
SR(k) = \lim_{t \to \infty} \frac{P(A > t) + P(B > t)}{1 - P(A \leq t, B \leq t)} = \lim_{t \to \infty} \frac{2(1 - p)}{1 - C(p, p)}
\]

where \( P(A > t) = P(B > t) = 1 - p \) and \( C(p, p) \) is the limit copula. One can calculate the failure measure if the copula is known. Kole (2006) discusses the use of different copulas in this context. Other alternative measures comprise the conditional probability of a specific failing institution, given the demise of another: \( P(A > t | B > t) \). With the methods developed below, the limiting value of this measure are rather straightforward to compute as well. But the point is that as the number of financials increases, the number of different partial measures one has to report rapidly increases. The \( SR(k) \), however, is good in any dimension, as it summarizes the systemic risk in a single measure.

In case that the risk is thin-tail distributed, the so-called Ledford–Tawn measure, provides more detailed information (by a logarithmic transformation of the probabilities); see Ledford and

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\(^3\) In practice one scales the different thresholds towards a common failure factor \( t \) by defining \( s = \theta t \) and \( z = \gamma t \). For the theoretical analysis this means that one can redefine the loss returns \( A \) and \( B \), by dividing these with the desired scales \( \theta \) and \( \gamma \) respectively. In the empirical analysis we look along the diagonal, i.e. take \( \theta = \gamma = 1 \).

\(^4\) Recall that it is straightforward to allow for different failure levels across firms.
Tawn (1997). In this case the SR(k) would just indicate asymptotic independence. But for the application to financials, the Ledford–Tawn measure is less informative than the SR(k) due to the fattailed nature of the risks.

2.2. Economic rationale for downside dependence

To provide a rationale for the downside dependence between banks and insurers, we start from an elementary factor model. The factors are assumed to follow a distribution with non-normal heavy tails. Firms and sectors partly differ with respect to their risk factors and this determines the differences in downside dependence within and between the sectors. The model is in the vein of De Vries’ (2005) portfolio approach to downside risk for banks.

The investments of banks and insurers are to a certain degree similar. Both invest in syndicated loans, have proprietary investments in equity and both hold mortgage portfolios. Moreover, financial instruments can transform insurance risk to financial investments (e.g., catastrophe bonds), or can transform default risk to insurance risk via credit default swaps. The securitization of bank loan portfolios widens the scope of investments for insurers.

There are also differences. Banks live from the interest rate spreads (intermediation margins), while life insurers receive premiums and have to pay the long interest rate. The deposit contract spreads (intermediation margins), while life insurers receive pre-

ments in equity and both hold mortgage portfolios. Moreover, financial investments (e.g., catastrophe bonds), or can transform
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ments in equity and both hold mortgage portfolios. Moreover, financial investments (e.g., catastrophe bonds), or can transform default risk to insurance risk via credit default swaps. The securitization of bank loan portfolios widens the scope of investments for insurers.

therefore, the results carry over to all distributions that exhibit power like behavior, as in

the normal distribution, that have all moments bounded due to the

Thus, the sum is almost entirely driven by the maximum of the observations.

\[
\lim_{t \to \infty} \frac{R(-t)}{R(-t)} = \lim_{t \to \infty} \frac{1 - R(-t)}{1 - R(t)} = t^{-2},
\]

for some \( x > 0 \).

For example, consider the case of the Student-t distribution with \( v \) degrees of freedom. Invoking L'Hôpital's rule, one can use the density \( r(t) \) to show that

\[
\lim_{t \to \infty} \frac{r(x) x}{r(x) x} = \lim_{t \to \infty} \left( \frac{1 + t^2 / v}{1 + t^2 / v} \right)^{(v+1)/2} = x^{-v}
\]

and hence \( x = v \). Furthermore,

\[
a = \lim_{t \to \infty} \frac{r(t)}{t^{v-2}} = \lim_{t \to \infty} t^{v-1} \left\{ I((v+1)/2) - \frac{\Gamma((v+1)/2)}{\sqrt{\pi v} \Gamma(v/2)} \right\} = \frac{1}{v} \frac{\Gamma((v+1)/2)}{\Gamma(v/2)} \frac{1}{\sqrt{\pi v}^{v-1/2}} x^{-v-1}.
\]

Thus for large \( t \)

\[
P(X > t) \approx at^{-v}.
\]

Define the re-scaled random variable \( Y = a^{1/2}X \), hence

\[
P(Y > t) = P(a^{1/2}X > t) = P(X > a^{-1/2}t) \approx t^{-v}.
\]

This gives an expression that is analogous to the pure Pareto case in (5), except for the fact that the Pareto expression is only good in the tails, as it only holds exactly in the limit

\[
\lim_{t \to \infty} \frac{P(Y > t)}{t^v} = 1.
\]

In the following, we investigate the downside dependence between two financials within a sector and across sectors. To this end, define the equity loss returns of a bank \( G_t \) and an insurer \( H_t \) as a portfolio of risk factors consisting of the following elements:

\[
G_t = F + B + Y_t \quad \text{and} \quad H_t = F + A + Z_t.
\]

For brevity we assume unitary scale coefficients for each factor, but this can be easily relaxed as we briefly discuss in the analysis that follows. In practice, the evidence is that the scales are considerably different across different companies, in contrast to the shape parameter \( x \); see Hyung and de Vries (2002).

2.2.1. Within-sector dependence

Consider first the dependency between two banks. The probability of a large loss \( t \) can be calculated with the help of Feller’s convolution theorem (1971, vol. II, Chapter VIII.8). Section A.1 of the Theoretical Appendix provides a brief exposition of this theorem. The Feller theorem holds that the probability of the sum of the marginal probabilities as \( t \) gets large. In other words, the probability mass along the axes above the portfolio line \( F + B + Y_t + t \) determines the convolution probability. Since the bank portfolio consists of three independent risk factors, the probability of a crash of a bank is

\[
P(F + B + Y_t > t) = 3t^{-v} + o(t^{-v}).
\]

Note that in case the scale factors differ from unity and be equal to \( f, b \) and \( y_t \) say, the result would just be

\[
P(F + B + Y_t > t) = (f + b + y_t)t^{-v} + o(t^{-v}).
\]

Thus, differences in scale only imply a quantitative difference.

But in case that the shape parameter \( x \) differs across the factors, this induces a qualitative difference. For example, suppose that the
idiosyncratic factor $Y_i$ has a higher shape parameter $\alpha_i > \alpha$ than the other two factors. Then
\[ P(F + B + Y_i > t) = 2t^{-2} + o(t^{-2}). \]

since the idiosyncratic factor contributes terms which are of smaller order (tend to zero faster). The rule of thumb is therefore that one can ignore factors that have thinner tails (higher shape parameters). The interested reader can easily adjust the analysis below for such cases. The empirical analysis automatically takes care of this possibility (since the extremes contributed by the other factors will dominate in the data).

Next, we determine the probability that two banks crash simultaneously
\[ P(G_1 > t, G_2 > t) = P(F + B + Y_1 > t, F + B + Y_2 > t). \]

To determine this systemic failure probability, we can again make use of Feller’s convolution theorem. Recall that the theorem holds that only the probability mass along the axes counts. The intersection of the two sets determined by the portfolio inequalities $F + B + Y_1 > t$ and $F + B + Y_2 > t$ hold simultaneously. Thus, the intersection only has points above $t$ along the $F + B$ axis in common. Note that the sum $F + B$ can be treated as a single random variable $Z$. Points along the $Y_1$ or the $Y_2$ axes larger than $t$ cannot simultaneously satisfy both equalities (e.g., in the three dimensional space $(Y_1, Y_2, Z)$, the point $(0,2t,0)$ satisfies $Z + Y_2 > t$, but not $Z + Y_1 > t$). This implies that
\[ \lim_{t \to \infty} P(F + B + Y_1 > t, F + B + Y_2 > t) = 1. \]

It follows that
\[ P(G_1 > t, G_2 > t) = P(F + B > t) + o(t^{-2}) = 2t^{-2} + o(t^{-2}). \] \tag{8}

The probability of a joint crash among two insurers is similar, thus $P(H_1 > t, H_2 > t) = 2t^{-2} + o(t^{-2})$.

2.2.2. Cross-sector dependence

Since the sector risk for the two companies is different, there are less common components in the portfolio of the two firms. The probability of a joint crash of an insurer and a bank is entirely determined by the single common factor $F$. If the portfolio inequalities $F + B + Y_1 > t$ and $F + A + Z_1 > t$ hold simultaneously, there is only probability mass of order $t^{-2}$ above $t$ along the $F$ axis in common, and no mass of this order along the $B$ and $A + Z_1$ axes. This implies that
\[ P(G_1 > t, H_1 > t) = P(F + B + Y_1 > t, F + A + Z_1 > t) = t^{-2} + o(t^{-2}). \] \tag{9}

2.2.3. Systemic risk

On the basis of (7)–(9) we can evaluate our measure for systemic risk or downside dependence (1). To this end, recall
\[ 1 - P(A < t, B < t) = P(A > t) + P(B > t) - P(A > t, B > t). \] \tag{10}

Combining (7), (8) and (10) one obtains the within-sector systemic risk \(4\) as
\[ SR(k) = \lim_{t \to \infty} \frac{P(G_i > t) + P(G_j > t)}{1 - P(G_i \leq t, G_j \leq t)} = \frac{3t^{-2} + 3t^{-2} - 2t^{-2}}{3t^{-2} + 3t^{-2} - 2t^{-2}} = \frac{6}{4}. \] \tag{11}

In words, the $SR(k)$ value indicates that in one out of the two cases when there is a bank failure, the other bank fails as well. Note that differences in scale would only qualitatively change this result, i.e. $SR(k)$ would still be higher than one. Different shape parameters, however, may also have a qualitative effect (for example, if both the idiosyncratic factors have larger shape parameters than $F$ and $B$, then $SR(k)=2$, i.e. the case of maximal asymptotic dependence; while in the opposite case that these are smaller, $SR(k)=1$).

The cross-sector systemic risk is found analogously from (7), (9) and (10)
\[ SR(k) = \lim_{t \to \infty} \frac{P(G_i > t) + P(H_j > t)}{1 - P(G_i \leq t, H_j \leq t)} = \frac{6}{5}. \] \tag{12}

The conditional expectation is higher in the case of within-sector dependence than in the case of cross-sector dependence, since the sectoral risks differ. In the empirical section, we estimate the systemic risk measure and test for the predicted difference. If the within-sector and cross-sector dependencies turn out to be equal, that would indicate that the sector risks are similar or unimportant; if these differ, that would vindicate the above sectoral factor structure.

2.2.4. Dependence and the normal distribution

It is interesting to note that the dependence in the tail disappears if we assume that the independent factors, $A, B, F, Y_1$ and $Z_1$ are normally distributed. Note that normality immediately implies that $G_i, G_j, H_i$ and $H_j$ are all correlated. If we assume that the returns on the individual projects exhibit heavy tails as before, there is dependence in the tails and the expected number of failures (1) converges to a number larger than one as the failure level $t$ increases. Even though there is positive correlation if the returns of both $G_i$ and $H_i$ follow a bivariate normal distribution, all dependence between the firms disappears as $t$ increases. Thus, under normality
\[ SR(k) = \lim_{t \to \infty} \frac{P(G_i > t) + P(G_j > t)}{1 - P(G_i \leq t, G_j \leq t)} = \lim_{t \to \infty} \frac{P(G_i > t) + P(H_j > t)}{1 - P(G_i \leq t, H_j \leq t)} = 1. \]

The proof for this result is similar to the proof of Proposition 2 in de Vries (2005) and follows directly from the general result by Sibuya (1960). This explains why Fig. 1a differs so much from Fig. 1b, especially in the northeast and southwest corners. The disappearance of the dependency in the tail area is not unique for the normal distribution. The same holds for exponentially distributed factors; see de Vries (2005). In the empirical section, we compare the semi-parametric estimate of (1), which allows for heavy tails, with the parametric estimates based on the bivariate normal model for the returns.

2.3. Effects of mergers

Lastly, we investigate how mergers affect the systemic failure probability. We consider a bilateral merger within a sector and across sectors. First, we consider an economy comprised of just two financial firms. Then, we consider an economy comprised of two banks and two insurers respectively.

In an economy with just two banks or two insurers, we have from (8) the systemic failure probability of order $2t^{-2}$. Suppose that the threshold failure level for a merged firm increases commensurately with its portfolio size. In that case, the systemic failure probability increases to
\[ P(G_i + G_j > 2t) = P(2F + 2B + Y_1 + Y_2 > 2t) = P(F + B + 2t\frac{1}{2}Y_1 + 2t\frac{1}{2}Y_2 > t) = (2 + 2^{1-\alpha})t^{-2} + o(t^{-2}). \] \tag{13}

In a two firm economy, the systemic risk due to a merger is equal to the risk of failure of the new firm. Thus, (13) exceeds (8), specifically

\footnote{Use that $P(1/2Y_1 > t) = P(Y_1 > 2t) = 2^{-\alpha}t^{-2}$ and apply Feller’s convolution theorem.}
Through the diversification effect.

Moreover, the double weight on firm economy, as well as in the three firm economy, the only com-

throughout the system risk in the four firm economy with that of a three firm economy, where two out of the four firms may merge. We compare the evaluation of the effect of mergers on the systemic risk.

Note that if \( \alpha > 1 \) then \( 1 + 2^{-\alpha} > 1 + 2^{-\alpha} \), so that a within-sector merger increases systemic risk by less than a cross-sector merger, due to the fact that the systemic risk among a separate bank and insurer is lower than among two separate banks (or insurers). However, also note that

\[
\lim_{t \to \infty} \frac{P(G_1 + H_1 > 2t; t, H_1 > t)}{P(G_1 > t, H_1 > t; t, H_1 > t)} = 1.
\]

The estimation of (1) can thus be reduced to the estimation of two univariate probabilities. The probabilities in the numerator and denominator can be easily estimated by counting the number of minima and maxima that exceed the threshold \( t \). Our count estimator thus reads

\[
\hat{E}[K|K \geq 1] = 1 + \frac{\#(\min\{A, B\} > t)}{\#(\max\{A, B\} > t)}.
\]

In the applications, we take \( t = 0.075 \) (i.e. a 7.5% loss return on a single day) in (18), close to the boundary of the sample, and we count the number of realizations of the min and max series that are above this threshold.\(^6\) The 7.5% loss return is close to the fifth highest loss return; see Table B.2, with summary statistics in the Empirical Appendix.

Next, we discuss the asymptotic properties of the appropriately scaled estimator. Divide both the numerator and denominator in (18) by the sample size \( n \). This turns the numerator and denominator into correlated U-statistics; see Serfling (1980, Chapter 5), since in this way one averages the excesses of the maxima and minima series. A Cramér delta argument applied to the ratio then yields the asymptotic normality as \( n \to \infty \) for fixed thresholds \( t \).

Subsequently, from statistical EVT it follows that one may let \( t \to \infty \) provided that this happens not too fast, i.e. such that \( M/n \to 0 \) and where \( M = \#(\max\{A, B\} > t) \), cf. De Haan and Ferreira (2006, p.260), or Huang (1992). Alternatively, one can apply the asymptotically normally distributed tail probability estimator from De Haan and Ferreira (2006, th. 4.4.7) for the numerator and the denominator in (18) and again invoke the Cramér delta method to establish asymptotic normality for the ratio as \( t \to \infty \).

To gain insight into the estimator (18), we conduct a small simulation experiment. To this end, we generate two series with 3120 draws (the number of observations in the real data) of pseudo-random variables from the standard normal and Student-t distribution with 3 degrees of freedom. Both series are re-scaled to give the same means, variances and correlation pattern as in the ABN AMRO and AXA series from Fig. 1a.\(^9\)

In Fig. 2, we plot the ratio of the number \( \min\{A, B\} > t \) to max \( \{A, B\} > t \) by varying the threshold \( t \). The thresholds are the order statistics from the two series; the x-axis gives the indices from the descending order statistics. The y-axis gives \( E[K|K \geq 1] - 1 \) from (2), plotted against the increasing rank order of the descending ordered statistics. As the rank along the x-axis increases, we move into the center of the sample and obtain more pairs with maxima and minima that exceed the threshold order statistic. For any finite sample, eventually \( E[K|K \geq 1] - 1 \) equals 1 at the lowest threshold \( t \), when \( t \) equals the smallest order statistic. But this is not the relevant area, since \( \text{SR}(\xi) = \lim_{t \to \infty} E[K|K > 1] \) should be judged from using a low number of order statistics only. Hence, the plots are based only on the first 750 descending order statistics from the combined series (where the choice for 750 is somewhat arbitrary).

The right-hand panel in Fig. 2 shows the result for the normal distribution. The plot first lingers at zero and then gradually moves upward. Since the normal distribution implies that all dependency vanishes asymptotically, the plot first remains close to zero and only then gradually increases.

The left-hand panel in Fig. 2 displays the result for the correlated Student-t series. This plot differs markedly from the correlated normal-based plot. Almost immediately, the series jumps to a level around 0.2 at which it stabilizes after some gyrations. Since far out in the tail areas there are just a few observations for which

\[^6\] Note that the loss returns are positive numbers, after multiplying the returns with \(-1\), appearing in the first quadrant.

\[^9\] The standard deviations are 0.019 and 0.023 for ABN AMRO and AXA respectively. The correlation coefficient is 0.575. The means are less than \( 10^{-2} \) in absolute value.
min \[A,B\] > t, the estimator (18) is initially unstable. But it rapidly settles around 0.2. This is indeed the level that one expects to see on the basis of calculations analogous to the calculations used to derive (11) and (12). In particular, for the Student-t series with 3 degrees of freedom (ignoring the means), one derives $SR(\kappa) - 1 = \frac{p^3}{(1 - \rho^3)^{3/2} + (\sigma_1/\sigma_2)^3} = 0.17106$.

Next, we turn to actual data. Using the same data as in the crossplot of the stock returns of AXA and ABN–AMRO in Fig. 1, Fig. 3 gives the results for the estimate of systemic risk (2). The results are very similar to those of the Student-t simulation in Fig. 2. On the left side of the graph, the plot is initially quite variable due to a lack of observations in the tail area, but it quickly stabilizes around a level of 0.28. Since, in our simulation, we choose the degrees of freedom equal to the power $a$ that is observed in the real data and set the correlation equal to the correlation that is observed between the AXA and ABN–AMRO returns, it is not so surprising that the estimator (18) stabilizes at a similar level as in the simulation experiment for the Student-t. The more important feature, though, is the fact that the plot immediately jumps to this level and does not gradually increase from zero, as it would if the data are asymptotically independent, like in the case of the normal distribution.

Below, we also investigate the robustness of our procedure by varying the threshold $t$. It is shown that the estimates do not change much, which is the force of statistical EVT. Moreover, in the Empirical Appendix, we construct confidence bands by the Jackknife re-sampling procedure.

### 4. Empirical results

We present the estimates of the systemic risk within and across the banking and insurance sectors.

### 4.1. Data

Our sample consists of the 10 largest European banks and the 10 largest European insurers. These firms are selected on the basis of balance sheet criteria, such as the amount of customer deposits and life and non-life premium income. Insurers can provide both life insurance and non-life insurance (e.g., property and casualty insurance). We use daily data from January 1992 until December 2003. The Appendix provides a precise description of the dataset. From the daily price quotes we construct the daily loss returns that are the empirical counterparts of $G_i$ and $H_j$ from the theory section. Table B.1 summarizes our classification of a financial intermediary as a bank or insurer and Table B.2 gives summary statistics of the loss returns.

### 4.2. Systemic risk estimates

We estimate the within-sector and cross-sector downside dependencies by means of (18), using $t = 0.075$. In other words, the threshold for the systemic risk is a 7.5% loss return on a single day. Since we have 10 banks and 10 insurers in our dataset, we have results for 45 possible combinations of banks, 45 possible combinations of insurers and 100 possible combinations between banks and insurers. Table 1 summarizes the estimation results for the 190 different combinations in total. The results for all 190 pairwise combinations are reported in Tables B.3, B.4 and B.5 in the Empirical Appendix, as well as confidence bands based on the Jackknife re-sampling procedure.

The summary results in Table 1 give the average and the median of all the $SR(\kappa) - 1$ estimates. Recall that $SR(\kappa) - 1$ is an estimate of the conditional joint failure probability (2). These results clearly indicate that the cross-sector dependence between banks and insurers is lower than the dependence between two firms from within the same sector. The average probability that two banks crash, given that one crashes is 10.4%. For insurers, this probability is similar and equals 11.7%. The probability that an insurer crashes, given that a bank crashes or that a bank crashes, given that an
insurer crashes is only 7.4%, however. In other words, while two banks on average jointly fail one out of every 9.5 times that there is a bank failing, a bank and an insurer fail jointly only one out of every 13.5 times that an insurer or a bank fails. It appears that the dependence is lower across the sectors.

To investigate the robustness of these conclusions and to show that this is not the result of sampling inaccuracy, we re-estimate (18) at the lower threshold \( t = 0.07 \). The results are collected in Table 2. While the averages and medians are somewhat different, the qualitative ranking is the same. Again, the cross-sectoral dependence is lower than the within-sector dependence. Moving into the sample by lowering the threshold \( t \) to 0.07 of course produces somewhat higher estimates. But, as we show below, under the presumption of normality, the estimates are much lower.

We formally test whether the cross-sector dependence is dissimilar from the dependence within the same sector by applying the Wilcoxon–Mann–Whitney signed ranks test to the Tables B.3,B.4,B.5.\(^ {11} \) The null hypothesis is that estimates from two of the three tables are coming from the same distribution. The alternative hypothesis is that the values differ.

We find that the probability that the banking sector dependencies are similar to the cross-sectoral dependencies is only 0.004%. We conclude that the risk profile of the two groups differs significantly. Using the same test procedure, we also find that the probability that the downside risk for combinations of insurers is equal to combinations of banks and insurers is only 0.003%. Thus, the dependence between banks and insurers is also significantly lower than the dependence among insurers. Moreover, the same test indicates that equality between the sectoral medians of banks and insurers is also not supported. But the rejection is less strong, as the difference between the sectoral medians is smaller than the difference with the cross-sectoral median.

On the firm level, there are sizable deviations from the average risk within the sector. Results for specific combinations of firms are given in Tables B.3, B.4 and B.5 in the Empirical Appendix. The largest conditional probability of a crash of two firms is 37.5% and involves two Spanish banks (Table B.3). The 37.5% is way above the sector average of 10.3%. A possible explanation for this high probability are the common exposures of the two Spanish banks (housing exposures, government bond investments) to risks in Spain and Latin America.

We also estimate (17), assuming a bivariate normal distribution for the returns. Tables B.7, B.8 and B.9 present the results, while Table 3 provides a summary. The results indicate again that the dependence between banks and insurers is also lower than the dependence among other combinations, lending robustness to our main conclusion. The order of magnitudes, though, are quite different. The assumption of normally distributed returns considerably underestimates the downside risk, both for the marginal and the multivariate risks. The average cross-sector systemic risk on basis of the normality presumption is so low that only in one out of every 158 times that there is a failure, both the insurer and the bank are expected to fail jointly. While the count measure says that this joint failure happens approximately once per 13 instances of a failure. A comparable huge difference regards the

\( \begin{array}{c|c|c|c|c}
\text{Table 1} \\
\multicolumn{5}{c}{\text{Summary non-parametric estimation results for } t = 0.075.} \\
\hline
& \text{Mean} & \text{Median} \\
& \text{Bank} & \text{Insurer} & \text{Bank} & \text{Insurer} \\
\hline
\text{Bank} & 0.1038 & 0.0744 & 0.095 & 0.069 \\
\text{Insurer} & 0.0744 & 0.1170 & 0.069 & 0.107 \\
\hline
\end{array} \)

\( \begin{array}{c|c|c|c|c}
\text{Table 2} \\
\multicolumn{5}{c}{\text{Summary non-parametric estimation results for } t = 0.07.} \\
\hline
& \text{Mean} & \text{Median} \\
& \text{Bank} & \text{Insurer} & \text{Bank} & \text{Insurer} \\
\hline
\text{Bank} & 0.1150 & 0.0864 & 0.0968 & 0.0842 \\
\text{Insurer} & 0.0884 & 0.1314 & 0.0842 & 0.1190 \\
\hline
\end{array} \)

\( \begin{array}{c|c|c|c|c}
\text{Table 3} \\
\multicolumn{5}{c}{\text{Estimation results (bivariate normal model).} } \\
\hline
& \text{Mean} \\
& \text{Bank} & \text{Insurer} \\
\hline
\text{Bank} & 0.0082 & 0.0063 \\
\text{Insurer} & 0.0063 & 0.0133 \\
\hline
\end{array} \)

\(^ {11} \text{We opt for this comprehensive test statistic given the limited amount of data. Alternatively, one can base oneself on the asymptotic normality of the individual pairwise estimates and use a Bonferroni bound. This approach is, however, overly conservative.} \)
within insurance sector joint failure probability. The normal based estimate shows this happens once per 75 times there is an insurer that crashes, which is way below the count measure estimate of once per 8.5 times. These huge differences are caused by the fact that under the assumption of joint normality, all dependence in the tail area eventually disappears. The non-parametric based estimator (18), though, shows that this is not the case.

For specific combinations of firms we find similar considerable differences between the normal based results and the frequency based count measure. The conditional probability of a double crash for the combination of HSBC and RBS for example is 0.083, while the normal distribution based estimate is only 0.0044. Our measure therefore predicts that the conditional probability of a double crash is approximately 20 times higher for this combination than the normal based measure would make believe. For the pair, AVIVA and AEGON, the estimate based on normality gives 0.0134. This is a factor 8 lower than the non-parametric based estimate of 0.111. Thus, the normal based measure gives a completely different view of the likelihood of joint crashes, whereas the normal distribution based model would indicate that this is essentially a zero probability event.

The empirical section investigates the dependence between combinations of financials, both within a sector and across sectors. We find that downside dependence between a bank and an insurer is significantly different from the dependence structure between two banks or between two insurers. The average probability that two banks crash, given that one crashes is 10.3%. For insurers this probability is 11.7%. The probability that an insurer crashes given that a bank crashes, or that a bank crashes given that an insurer crashes, is only 7.4%. The latter figure is in line with the empirical evidence reported in van Leeuwen and Knot (2009) for European financial conglomerates. Moreover, it indicates that, in general, downside dependence is lower for cross-sector combinations. The theoretical model explains this by the fact that there are fewer common factors. But while such cross-sectoral mergers may reduce the risk of individual financial institutions if cross hedging at the holding level were allowed, mergers can at the same time increase the systemic risk. We showed, though, that a merger of a subset of the firms embedded in a larger economy may, to a first order, have no impact on the systemic risk, while there are four subsector individual firm benefits from the formation of a conglomerate.

The current drive to unwind bancassurance conglomerates must therefore be due to motives other than risk, such as management complexity and the separate pillar structure of the regulatory frameworks for bank and insurance activities. Schmid and Walter (2009) generally find that scope is not value enhancing, but also show that combinations of commercial banking and insurance are on balance positive. This time, due to the economy wide character of the 2007–2009 credit crisis, banks and insurance companies are both severely affected. This was different at the time of the bursting of the internet bubble, when insurers were hard hit, while banks came off lightly. Dissolving conglomerates now may therefore be a myopic error. The next crisis will surely be different. The recent financial crisis has shown once again that fat tails and strong dependency are real. Thus, higher capital requirements and full recognition of off-balance commitments in risk-weighted capital calculations are a necessity. Moreover, proper systemic risk evaluation requires aggregation across institutions and sectors, rather than the micro-based approach that only looks at individual institutions as in the Value at Risk methodology. Our conditional failure index is a measure that goes into the direction of making the much needed macro approach to the financial stability issue operational.

Acknowledgments

We are grateful to a referee for the excellent comments on a previous version and Linda van Goor for an insightful discussion. Furthermore, we like to thank conference and seminar participants at the DNB, the BIS–CEPR–JFI meeting, the VU University Amsterdam and the Bundesbank Dresden 2010 conference. The opinions expressed in this paper are those of the authors and do not necessarily represent the views of AEGON Asset Management.

Appendix A. Theoretical appendix

In this appendix, we review the Feller convolution result and the case of the normal distribution for the $SR(n)$ measure.

A.1. Feller’s theorem

We briefly introduce Feller’s convolution theorem (1971, VIII.8). This is needed to calculate convolutions of fat-tailed random variables. The convolution result is also used to determine the downside interdependence (systemic risk). The Feller theorem holds that if two independent random variables $A$ and $B$ satisfy (5)

$$P(A > t) = P(B > t) = t^{-\alpha},$$

then their convolution satisfies

$$\lim_{t \to \infty} \frac{P(A + B > t)}{2t^{-\alpha}L(t)} = 1,$$

and where $L(t)$ is slowly varying (i.e. $L(at)/L(t) \to 1$, for any $a > 0$). In other words, the theorem implies that for large failure levels $t$, the convolution of $A$ and $B$ can be approximated by the sum of the marginal distributions of $A$ and $B$. All that counts for the probability of the sum is the marginal probability mass that is located along the two axes above the points where the line $A + B = t$ cuts the two axes.

To show this, first note that since $A$ and $B$ are independently Pareto distributed

$$1 - P(A \leq t, B \leq t) = 1 - [1 - t^{-\alpha}]^2 = 2t^{-2\alpha} - t^{-2\alpha} \sim t^{-2\alpha}$$

as $\lim_{t \to \infty} (2t^{-2\alpha} - t^{-2\alpha}) t^{-2\alpha} = 2$. Since (for positive random variables)

$$P(A + B > t) \geq 1 - P(A \leq t, B \leq t),$$

we have the bound $P(A + B > t) \geq 2t^{-2\alpha}$. The Feller theorem maintains that $P(A + B > t)$ is in fact approximately $2t^{-2\alpha}$ as $t$ becomes large, i.e. is also the lower bound. To verify this, consider the probability

$$P(A + B > t) - \frac{1}{2} - P(A \leq t, B \leq t),$$

which comprises the probability mass in the triangle above the line $A + B = t$ (with vertices $(0, t)$, $(t, 0)$ and $(t, t)$). We argue that the probability mass in the triangle is of an order smaller than $t^{-2\alpha}$. Note that by independence, for $\alpha \in (0, 1)$
Thus, for any slab above the line \( A + B = t \) and with vertex at \((\lambda, 1 - \lambda)\) on the line \( A + B = t \), the probability mass is of an order smaller than \( t^{-2}\) (i.e. \( \lim_{t \to \infty} t^{-2}\sqrt{t^2 - 0} = 0 \)). This specific slab partly covers the triangle. By varying \( \lambda \), this shows that the entire triangle carries probability mass of an order smaller than \( t^{-2} \).

A.2. Normal case

Assume all factors \( A, B, F, Y_i \) follow a standard normal distribution. From the additivity properties of the normal distribution and using Laplace's asymptotic expansion, we have that

\[
P(G_i > t) = P(F + B + Y_i > t) = P(\sqrt{3F} > t) \\
\sim \frac{1}{2\sqrt{2}\pi} \sqrt{\frac{3}{t}} \exp\left(-\frac{1}{2} \frac{t^2}{3}\right)
\]

as \( t \to \infty \). Analogously,

\[
P(G_i + G_j > t) = P(2F + 2B + Y_i + Y_j > t) = P(\sqrt{10F} > t) \\
\sim \frac{1}{2\sqrt{2}\pi} \sqrt{\frac{10}{t}} \exp\left(-\frac{1}{2} \frac{t^2}{10}\right).
\]

Furthermore

\[
\frac{1}{P(G_i + G_j > 2t) - P(G_i > t)} \sim \frac{1}{2\sqrt{3\pi} \sqrt{\frac{3}{t}}} \exp\left(-\frac{1}{2} \frac{t^2}{3}\right) \sim \sqrt{\frac{10}{4\sqrt{3}}} \exp\left(-\frac{1}{30} \frac{t^2}{2}\right) \to 0
\]

as \( t \to \infty \). The following bound

\[
P(G_i > t) + P(G_j > t) \leq \frac{1}{1 - P(G_i + G_j > 2t)} \leq \frac{1}{1 - P(G_i + G_j > 2t)} \frac{1}{2\sqrt{2}\pi} \sqrt{\frac{3}{t}} \exp\left(-\frac{1}{2} \frac{t^2}{3}\right)
\]

therefore implies that

\[
\lim_{t \to \infty} \frac{P(G_i > t) + P(G_j > t)}{1 - P(G_i \leq t, G_j \leq t)} = 1.
\]

Appendix B. Empirical Appendix

In this appendix, we discuss the selection of the data and we give the detailed results for the pairwise downside risk estimates for the count measure \( \text{SR}(k) \) and under the assumption of normality.

B.1. Data selection

Since it is common for financial companies in Europe to exploit a broad portfolio of activities in banking and insurance, it is difficult to construct a dataset of companies pursuing pure banking or insurance strategies. Moreover, some activities as, for example, the provision of mortgages, are common for all companies in both banking and insurance. In this section we will explain when we define a company being a bank or an insurer.

We distinguish three different categories: banks, insurers (combining property & casualty and life insurance business) and financial conglomerates. The dataset contains companies from Europe (the EU and Switzerland). First, we take the largest firms by market capitalization in the following sectors from Datastream: banking, life insurance, insurance and other financial services. We classify these companies based on their annual accounts over 2002.

To be able to make a distinction between insurers and banks, we collect the following balance sheet items: ‘customer deposits’, ‘technical provisions’ and ‘life-insurance risk born by the policy holder’. We assume that these broad items are unique for specific sectors. The item ‘customer deposits’ is typical for banks, since they borrow money from the public. The item ‘technical provisions’ is typical for insurers, since it represents the size of provisions for future insurance claims. Another item typical for life insurance is ‘life-insurance risk born by the policy holder’, which represents provisions for future claims of life insurance policies. The three items are added up and we represent the customer deposits as a percentage of this sum of balance sheet items. When the percentage of deposits is larger than 90% we define a financial firm as a bank. When the sum of ‘technical provisions’ and ‘life-insurance risk born by the policy holder’ represented as a percentage of the sum of all three items is larger than 90%, we define the firm as an insurer. Table B.1 summarizes our classification for the different financial intermediaries.

We make a distinction between property and casualty insurers and life insurers and collect data on the net premium income of insurers. The net premiums are the gross premiums written minus reinsurance cover. Since an insurer may choose to buy reinsurance cover for some lines of business, we argue that the net premium income gives the best information as to whether an insurer is active in life insurance or in property and casualty insurance. The life-insurance premium income is represented as a percentage of the total premium income.

We use data from 1992 to 2003, since Basel I came into effect in 1992. Data is on a daily basis. Firms that are part of a larger conglomerate, like Winterthur which is a holding of Credit Suisse, are excluded. Some firms are omitted because the available data series is too short. Summary statistics of the loss returns are provided in Table B.2. For HSBC we lack a few observations at the start of the sample. But we prefer to keep the starting date in 1992 when Basel I came into effect. The missing data for HSBC do not hamper the empirical analysis.

Table B.1

<table>
<thead>
<tr>
<th>Insurer</th>
<th>Bank</th>
<th>Life</th>
<th>Non-life</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENERALI</td>
<td>0.00</td>
<td>1.00</td>
<td>0.65</td>
</tr>
<tr>
<td>AXA</td>
<td>0.00</td>
<td>1.00</td>
<td>0.70</td>
</tr>
<tr>
<td>AEGON</td>
<td>0.01</td>
<td>0.97</td>
<td>0.96</td>
</tr>
<tr>
<td>AVIVA</td>
<td>0.00</td>
<td>1.00</td>
<td>0.75</td>
</tr>
<tr>
<td>PRUDENTIAL</td>
<td>0.06</td>
<td>0.94</td>
<td>0.98</td>
</tr>
<tr>
<td>ZFS</td>
<td>0.00</td>
<td>1.00</td>
<td>0.30</td>
</tr>
<tr>
<td>LEGAL&amp;GENERAL</td>
<td>0.00</td>
<td>1.00</td>
<td>0.94</td>
</tr>
<tr>
<td>ALLENAZA</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>ROYALASUN</td>
<td>0.00</td>
<td>1.00</td>
<td>0.82</td>
</tr>
<tr>
<td>SKANDIA</td>
<td>0.08</td>
<td>0.92</td>
<td>0.99</td>
</tr>
</tbody>
</table>

12 For BBVA Datastream reported a value of 28,757 on December 26 1995, while the prices on the surrounding days hover between 172 and 173. For this day, which is second Christmas day in Europe, Datastream does not report quotes different from the last active trading day. Friday 22nd for any of the other companies that we use. The corresponding quotes on Bloomberg do not have differences until the second digit. For this reason we imputed a corrected price equal to the record on the first day of Christmas, as is done in Datastream for the other companies. Datastream has now imputed the corrected value after this was brought to their attention.
The largest realized daily loss of 24.6% is for Skandia, close to the largest loss of 24.3% realized by Royal & Sun. The 5th largest losses are already a bit smaller, respectively 12.9% for Skandia and 12.6% for Royal & Sun; but the 5th largest loss of ZFS is larger. The mean returns are positive, except for Royal & Sun. Standard deviations of the returns are very similar. Under the assumption of normality, the 5th largest loss returns are still close to their theoretical values of respectively 9.3% and 7.7% for Skandia and Royal & Sun. But the normal model fails for the largest loss of 24.3% realized by Royal & Sun. Standard deviations of the returns are very similar. Under independence, for example, the estimate of $\mathbb{E}[\kappa|\kappa \geq 1] > 1$ at the limiting failure levels. In practice, the estimate of $\mathbb{E}[\kappa|\kappa \geq 1]$ is evaluated at finite failure levels. Thus, in practice one estimates $\mathbb{E}[\kappa|\kappa \geq 1]$, but at high failure levels. One may wonder how much this matters. Under dependence, for example, $\mathbb{E}[\kappa|\kappa \geq 1]$ is greater 1, but at finite failure levels nevertheless $\mathbb{E}[\kappa|\kappa \geq 1] > 1$.13 The same observation holds for jointly normal distributed returns. To investigate this issue, we also estimate $\mathbb{E}[\kappa|\kappa \geq 1]$ under the assumption of independence for one pair of banks. We use the two Spanish banks singled out for discussion in the main text. From Table B.3, we have that BSCH and BBVA have a conditional joint failure probability of 37.5%. For these two banks the conditional estimate of $\mathbb{E}[\kappa|\kappa \geq 1]$ is considerably larger than the count-
In the applications we take $t = 0.075$ in (18) close to the boundary of the sample. The choice of the threshold is driven by the desire to take it relatively large, since this is the failure and systemic risk area, but this desire is tempered by the need to have sufficient data for estimation purposes. To establish confidence bands, we opt for the robust non-parametric Jackknife method, rather than relying on asymptotic theory with only 12 years of daily data, which is small if one studies bivariate dependence.

### Table B.4

<table>
<thead>
<tr>
<th>Combinations</th>
<th>Lower bound</th>
<th>Point estimate</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROYAL&amp;SUN-AEGON</td>
<td>1.194</td>
<td>1.225</td>
<td>1.257</td>
</tr>
<tr>
<td>AEGON-AVIVA</td>
<td>1.100</td>
<td>1.111</td>
<td>1.125</td>
</tr>
<tr>
<td>RBS–STD CHARTERED</td>
<td>1.056</td>
<td>1.091</td>
<td>1.100</td>
</tr>
<tr>
<td>BSCH–LEGAL&amp;GENERAL</td>
<td>1.063</td>
<td>1.063</td>
<td>1.071</td>
</tr>
</tbody>
</table>

Based measure 1.375. Both results therefore indicate that the way in which we measure $SR(j)$ gives answers that considerably differ from the cases of asymptotic independence.

### B.3. Confidence bands

The point estimate is estimated using the full sample. As a by-product, it becomes clear that the point estimates do not change much if we omit a sequence of observations.

### Table B.6

<table>
<thead>
<tr>
<th>Combinations</th>
<th>Lower bound</th>
<th>Point estimate</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
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<td>1.056</td>
<td>1.091</td>
<td>1.100</td>
</tr>
<tr>
<td>BSCH–LEGAL&amp;GENERAL</td>
<td>1.063</td>
<td>1.063</td>
<td>1.071</td>
</tr>
</tbody>
</table>

Table B.5 reports some of the Jackknife confidence bands for a number of $SR(j)$ estimates. A selection of the results is compiled in this table for considerations of space. The bounds of the confidence interval do not deviate considerably from the point estimates and are of the same order. The central column gives the point estimate from (18). In the left and right column one finds the 90% confidence interval. In the case of the combination of BSCH and Legal and General, the point estimate of (18) hits the lower bound. This is the result of the quite limited sample, of only 12 years of daily data, which is small if one studies bivariate dependence.

### Table B.7

<table>
<thead>
<tr>
<th>Combinations</th>
<th>Lower bound</th>
<th>Point estimate</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
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<td>1.225</td>
<td>1.257</td>
</tr>
<tr>
<td>AEGON-AVIVA</td>
<td>1.100</td>
<td>1.111</td>
<td>1.125</td>
</tr>
<tr>
<td>RBS–STD CHARTERED</td>
<td>1.056</td>
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<td>1.100</td>
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<tr>
<td>BSCH–LEGAL&amp;GENERAL</td>
<td>1.063</td>
<td>1.063</td>
<td>1.071</td>
</tr>
</tbody>
</table>

Thus, we can obtain a confidence band by the Jackknife resampling procedure. To this end the data are divided in 20 blocks of 156 observations. We then apply estimator (18) 20 times, each time leaving one block of 156 observations out of the time series. To obtain the confidence band, the highest and lowest estimation results are removed and the interval between the next highest and lowest statistics then provides the 90% confidence interval. The point estimate is estimated using the full sample. As a by-product, it becomes clear that the point estimates do not change much if we omit a sequence of observations.
B.4. Multivariate normal results

To put the count-based measure into perspective, we also calculate the systemic risk estimates under the assumption of joint normality. We start with the results for the banking sector in Table B.7. The next table, Table B.8, is the normal based systemic risk table for the insurers. The cross-sectoral results under the presumption of joint normality follow in Table B.9.

References


