A New Approach to The Complexity of Decision Problems

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Abstract

A new measure of the complexity of optimal economic decisions is introduced. It is based on the level of detail of information (no information; ordinal; and cardinal information) that is required to establish optimality. A detailed example involving sequential group decision making is provided. It is shown that the type of links between successive agents determines the degree of complexity. The measure is also illustrated in the realm of matching problems.

Keywords: Complexity, Group Decision Making, Bounded Rationality

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1 Introduction

This paper is concerned with the complexity of economic decision problems. Such concerns form one of the mainstays of recent formal approaches to bounded rationality, see for example

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Rubinstein (1998). The measure proposed in this paper is new. It is based on the level of detail of information (cardinal, ordinal, no information) that is required to determine the set of optimal solutions to a decision problem. It can be usefully applied to matching problems and collective or group decision problems.

In matching or assignment problems, the aim is to match two groups of entities—medical students and hospitals, marriageable men and women, workers and managers. As, say, both medical students looking for internships and hospitals differ in their characteristics (e.g., preferences over educational programs, responsibilities and salary offered for applicants; desired final grades or class standings for hospitals, see Roth (1984)), the value of the match is likely to depend on the precise match of the underlying characteristics. Suppose, following Becker (1973), that marriageable men and women differ in a trait, x and y, respectively, with $x_1 < \ldots < x_n$ and $y_1 < \ldots < y_n$, and that the value generating function $f(x,y)$ is complementary, $f_{xy} > 0$. Then in equilibrium matching is positively assortative. That is, likes are matched, or (x_i, y_i), $i = 1, \ldots, n$. Note that one only has to be able to *order* men and women on the basis of this trait. That is, ordinal information about the traits of men and women is sufficient (and necessary) to characterize this optimal matching. Kremer and Maskin (forthcoming), in their study of growing wage inequality and segregation by skill in the USA, use a matching function between employees with nonmonotonic marginal products, $f(x,y) = \max(xy^2, x^2y)$. Shimer and Smith (2000) note that for this production function "matching patterns are not easily characterized" (p. 346). Using the complexity measure introduced in this paper, I am able to indicate that this increased difficulty of characterizing value maximizing matches stems exactly from the fact that ordinal information is no longer sufficient. Instead, cardinal information is required as can be seen from a simple example. Suppose there are just two types of workers, high ability H and low ability $L < H$, and suppose there are two workers of each type. It would be value maximizing for high ability workers and low ability workers to segregate in separate firms instead of forming mixed firms if and only if $H^3 + L^3 > 2LH^2$ or $H > \frac{1}{2}(1+\sqrt{5})L$. Clearly, to verify which matching pattern is best, ordinal information about the characteristics of the employees ("H is larger than L") is no longer sufficient, and this is the reason the assignment problem is more complicated.

This example suggests that a useful classification of matching problems in terms of their complexity can be based on the required degree of detail of information (cardinal; ordinal; no information) regarding the entities that are to be matched. Such a measure may be thought to give a first, coarse and qualitative classification that can be further refined. In the above example there are just two ability levels. Adding a third, intermediate ability level further complicates the matching problem but this would not show up in the measure of complexity proposed here.

In the next section I show how a measure of complexity based on the varying informational demands imposed on an organizational designer can be used to classify simple sequential group decision structures. It will be shown that, depending on the type of links between successive decision makers, ensuring optimality requires no information, ordinal information, or cardinal information about the qualities of the individual decision makers. The third section concludes by comparing the present approach to complexity with other approaches that can be found in the literature.

2 An Example

2.1 The Model

As the model in this section is merely used to illustrate how one can classify decision structures in terms of their degree of complexity, I use the simplest model I am aware of, the one introduced by Sah and Stiglitz (1986).

Assume there is a project that can be either of good quality, $q = g$ (which is the case with probability α) or of bad quality, $q = b$. If implemented, a good project gives rise to profit X while a bad project leads to a loss Y.

An agent $i \in I = \{1, \ldots, n\}$ can either accept, A, or reject, R, a project. Ideally, one would like the agents to accept all good projects and to reject all bad projects. However, following Sah and Stiglitz (1986) I assume that every agent $i \in I$ is fallible: $p_i^g < 1$, *i.e.*, the probability with which agent i accepts a good project is smaller than one, and $p_i^b > 0$, or the probability with which agent i accepts bad projects is larger than zero. Agent i will be called better than j if the former accepts more good projects, $p_i^g > p_j^g$, and less bad projects, $p_i^b < p_j^b$. Such a situation will be denoted by $i \succ j$.

Assumption 1 *The agents $i \in I$ are ordered:* $1 \succ 2 \succ 3 \succ \ldots \succ n$.

This assumption excludes discussion of situations in which an agent i accepts both more good *and* more bad projects than agent j. Moreover, the possibility of identical agents is ignored. Let T be the set of profiles of $\{(p_1^g, p_1^b), \ldots, p_n^g, p_n^b)\}$.

One interpretation, offered by Sah and Stiglitz (1986), of the behavioural assumption is that agents receive a binary signal and have a decision rule that is exogenously fixed and does not depend on their position in the decision structure. As this paper is concerned with the introduction of a measure of complexity, I feel that the simplicity of this assumption is acceptable. It is the simplest model I am aware of that allows for heterogeneity of agents. It would certainly be worthwhile to study the complexity of decision structures in the presence of richer behavioural assumptions. Alternatively, one could view the agents as tests that are run by various departments to check the viability of a product. The errors they make are then simply statistical type I and type II errors, and it seems quite plausible that the quality of individual tests does not depend on tests run by other parts of the organization.

Now think of an "organizational designer" who has got on the one hand a group of error–prone employees, the qualities of whom she may or may not know, and, on the other hand, a decision structure containing positions to which she wants to assign these employees. She wants to answer the question whether it is possible to know *ex–ante* on the basis of the possibly limited knowledge she possesses about the capabilities of her employees whether she can correctly assign them to the positions in the decision structure. Employees will be said to have been correctly assigned if changing their positions does not increase the expected profit the decision structure generates.

A decision procedure specifies both a structure Σ and an assignment ϕ.

Definition 1 *A* decision procedure *is a pair* (Σ, ϕ). *The* decision structure Σ *is a finite, directed, rooted binary tree. An* assignment $\phi : I \to \Sigma$ *is a mapping from the set of agents* I *to the set of positions* Σ. *Let* $\mathcal{S}$ *be the set of structures.*[1]

Structures that satisfy Definition 1 will be called *admissible* structures. An example of an admissible decision structure is shown in Figure 1. The nodes stand for organizational

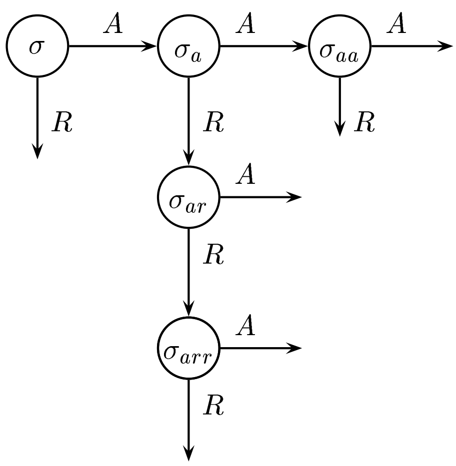


Figure 1: A decision structure Σ

departments, bureaus or desks and the directed edges represent the direction of flow of projects. The label on an edge starting at a node is associated with the action taken at that node. Carter (1971) shows various tree-like decision structures that resemble the ones studied here.

Since a structure is a binary tree, every node σ can be reached by just one, finite, ordered

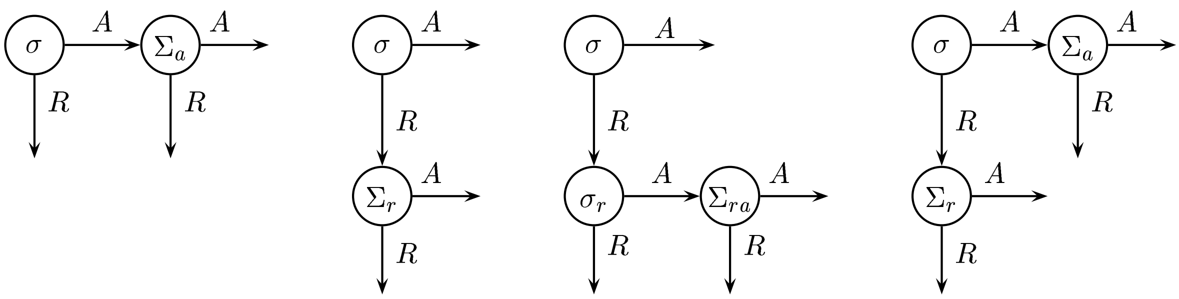
[1] Note that the assignment is a mapping from agents to nodes, not from nodes to agents. This implies no limitation in the framework studied here as I exclude the possibility of identical agents. Note moreover that one needs to increase the number of agents with the number of positions.

series of accept and/or reject decisions. Every node will be indexed by this series of decisions. For example, a node that is reached after an acceptance and a subsequent rejection, will be denoted by σ_{ar}. The root is denoted by σ. That part of the structure that starts with the node σ_{aa} is itself a structure and will be called a sub–structure. It will be indexed by the unique series of decisions through which it can be reached: Σ_{aa}. It will be useful to let j, l and k stand for a finite series of a's and r's. The symbol σ_j may stand for the root σ and analogously Σ_j may denote the whole structure Σ.

For every pair of nodes (σ_j, σ_l), let ω_{jl} denote the first common predecessor of σ_j and σ_l. Graphically, this is the first node that is both on the path back from σ_j to the root σ *and* on the path back from σ_l to the root. $\Sigma(\omega_{jl})$ is the *substructure* that starts with ω_{jl}. I call this the smallest substructure that contains both σ_j and σ_l.

Example In Figure 1, $\omega_{ar,aa} = \sigma_a$, and therefore the sub–structure $\Sigma(\omega_{ar,aa})$ equals Σ_a.

Since in what follows all admissible decision structures will be classified, the basic building blocks with which one can build these admissible structures recursively have to be introduced. Figures 2 (a), (b) and (d) are useful in this respect.



(a) hierarchical (b) polyarchical (c) hybrid (d) omniarchical

Figure 2: Building blocks (a), (b) and (d). Hybrid (c)

Definition 2 *There are three building blocks. The first is a* hierarchical building block, *consisting of a node σ, a generic sub–structure Σ_a connected to σ by an arrow labeled A. This is illustrated in Figure 2 (a). Such a connection will be denoted by $\sigma\mathcal{H}\Sigma_a$. A* polyarchical building block *is the second building block: it consists of a node σ, a generic sub–structure Σ_r connected to σ by an arrow labeled R. It is depicted in Figure 2 (b). This connection will be denoted by $\sigma\mathcal{P}\Sigma_r$. Finally, an* omniarchical building block *consists of a node σ, a sub–structure Σ_a connected by an edge labeled A to σ, and a sub–structure Σ_r connected to σ by an edge with label R. This building block will be denoted by $\mathcal{O}(\sigma, \Sigma_a, \Sigma_r)$ and is illustrated in Figure 2 (d).*

The distinguishing feature of the omniarchical building block (or of an omniarchy for that matter) is the presence of a node, σ, that cannot make a final decision: whether the agent at σ decides to reject or to accept a project his decision will not be final, the project will always be screened by yet another agent. The role of this agent is to allocate projects to either the substructure Σ_a or Σ_r.

Using these three building blocks, one can construct any admissible decision structure recursively by "filling in" the appropriate substructure(s) with one of the above three building blocks.

Example (continued) The structure in Figure 1 is uniquely described by the expression $\Sigma = \sigma\mathcal{H}\mathcal{O}(\sigma_a, \sigma_{aa}, \sigma_{ar}\mathcal{P}\sigma_{arr})$.

A structure that contains only hierarchical (polyarchical) building blocks is called a *pure hierarchy (pure polyarchy)*. A *pure structure* refers to either of these. A *hybrid structure* is made up of both hierarchical and polyarchical building blocks. A simple example is provided in Figure 2 (c). Formally, it can be described as $\sigma\mathcal{X}\sigma_i\mathcal{X}\ldots\sigma_l$, with $\mathcal{X}$ equal to either $\mathcal{H}$ or $\mathcal{P}$. A structure will be called *dual* if it contains at least one omniarchical building block.[2]

The probability with which a decision procedure (Σ, ϕ) accepts a project of quality q is

[2] I use the terms hierarchy and polyarchy only to remain in line with Sah and Stiglitz (1986). Both hierarchical and polyarchical decision structures as defined here can be part of real world hierarchies.

denoted by $p^q(\Sigma,\phi)$. The structure Σ determines the form of the mathematical expression, whereas the assignment ϕ fixes the specific values of the variables of the expression. Exploiting the fact that Σ can be recursively constructed, the form $p(\Sigma)$ of $p^q(\Sigma,\phi)$ can be defined as follows:

$$p(\Sigma) = \begin{cases} p(\sigma)p(\Sigma_a) & \text{if} \quad \Sigma = \sigma\mathcal{H}\Sigma_a \\ p(\sigma) + (1-p(\sigma))p(\Sigma_r) & \text{if} \quad \Sigma = \sigma\mathcal{P}\Sigma_r \\ p(\sigma)p(\Sigma_a) + (1-p(\sigma))p(\Sigma_r) & \text{if} \quad \Sigma = \mathcal{O}(\sigma,\Sigma_a,\Sigma_r) \end{cases} \tag{1}$$

Example (continued) Consider the example of $\Sigma = \sigma\mathcal{H}\mathcal{O}(\sigma_a,\sigma_{aa},\sigma_{ar}\mathcal{P}\sigma_{arr})$ depicted in Figure 1. Its probability function equals

$$p(\Sigma) = p(\sigma)\left[p(\sigma_a)p(\sigma_{aa}) + (1-p(\sigma_a))\left(p(\sigma_{ar}) + (1-p(\sigma_{ar}))p(\sigma_{arr})\right)\right] \tag{2}$$

The assignment ϕ determines how agents are assigned to positions. For a given structure let Φ be the set of assignments of agents to nodes.

Example (continued) Consider the structure $\Sigma = \sigma\mathcal{H}\mathcal{O}(\sigma_a,\sigma_{aa},\sigma_{ar}\mathcal{P}\sigma_{arr})$ and suppose that the assignment of agents $(1,2,3,4,5)$ equals $\phi(1)=\sigma$, $\phi(2)=\sigma_a$, $\phi(3)=\sigma_{aa}$, $\phi(4)=\sigma_{ar}$ and $\phi(5)=\sigma_{arr}$. The probability with which organization (Σ,ϕ) accepts a project of good quality then becomes

$$p^g(\Sigma,\phi) = p_1^g\left[p_2^g p_3^g + (1-p_2^g)(p_4^g + (1-p_4^g)p_5^g)\right] \tag{3}$$

The expected profit of an organization (Σ,ϕ) equals

$$E(\Pi;\Sigma,\phi) = \alpha X p^g(\Sigma,\phi) - (1-\alpha)Y p^b(\Sigma,\phi) \tag{4}$$

Let $C : \mathcal{S}\times T \to \Phi$ be the correspondence such that $C(\Sigma,t)$ is the set of optimal assignments given the structure Σ and profile t.

In what follows, use will be made of so–called pair–wise switches. By a pair–wise switch I mean a switch of the position of a pair of agents i and j, initially located at σ_l and σ_m, respectively, in a given structure Σ. This pair–wise switch leaves the position of all other agents unaffected. The switch may or may not change the expected profit of the organization, $\Delta E(\Pi; \sigma_l, \sigma_m)$:

$$\alpha X p^g(\sigma \to \omega_{l,m}) \psi^g(\sigma_l, \sigma_m) - (1-\alpha) Y p^b(\sigma \to \omega_{l,m}) \psi^b(\sigma_l, \sigma_m) \tag{5}$$

where

$$\psi^q(\sigma_l, \sigma_m) := (p_i^q - p_j^q) f^q(\Lambda \backslash \sigma_l, \sigma_m) \tag{6}$$

for $q \in \{g, b\}$.

Before providing an example, let me discuss the components of Equations 5 and 6. The probability with which a project reaches $\omega_{l,m}$ equals $p^q(\sigma \to \omega_{l,m})$. The term $p_i^q - p_j^q$ is the difference in probability with which the agents that are being switched accept a project. This is multiplied by $f^q(\Lambda \backslash \sigma_l, \sigma_m)$, a term that takes as its arguments the probabilities of acceptance of whoever has been allocated to nodes in the set $\Lambda \backslash \sigma_l, \sigma_m$. This *set* consists of $\omega_{l,m}$, the nodes on the paths connecting $\omega_{l,m}$ with σ_l and σ_m, respectively, and any nodes that succeed either σ_l or σ_m. Neither σ_l nor σ_m, the nodes where agents i and j are positioned, are part of this set. The multiplicative relationship between $p_i^q - p_j^q$ involving agents i and j located at nodes σ_l and σ_m, and $f^q(\Lambda \backslash \sigma_l, \sigma_m)$, a set of nodes excluding both σ_l and σ_m, stems from the nature of the admissible structures and the fact that no agent appears more than once in the organization.

It is crucial to distinguish (i) the set of nodes $\Lambda \backslash \sigma_l, \sigma_m$ and the function $f^q(\cdot)$ defined on it from (ii) the substructure $\Sigma(\omega_{l,m})$ and the related function $p^q(\Sigma(\omega_{l,m}))$. The latter function is a probability function, whereas $f^q(\cdot)$ is not. Instead, it is merely a function of the characteristics of the agents located at nodes in the set $\Lambda \backslash \sigma_l, \sigma_m$.

Example (continued) Suppose $\phi(1) = \sigma$, $\phi(2) = \sigma_a$, $\phi(3) = \sigma_{aa}$, $\phi(4) = \sigma_{ar}$ and $\phi(5) = \sigma_{arr}$, and suppose agents 3 and 4 are switched. Then $p^q(\sigma \to \omega_{aa,ar}) = p_1^q$, and

$$f^q(\Lambda \backslash \sigma_{aa}, \sigma_{ar}) = p_2^q + p_5^q(1 - p_2^q) - (1 - p_2^q) \tag{7}$$

Clearly, this is not the probability with which some admissible decision structure accepts a project. ◇

Although the agents $i \in I$ are ordered, the organizational designer may not have the required level of detail of information to determine the optimal assignment. For a given Σ, say that

1. no information is needed to determine the optimal assignments if $C(\Sigma, t)$ is the same for all $t \in T$;

2. ordinal information is needed to determine the optimal assignments if, the preceding condition does not hold, but for any two profiles t and t' such that the agents have the same ordinal ranking, $C(\Sigma, t) = C(\Sigma, t')$;

3. otherwise cardinal information is needed to determine the optimal assignments.

These informational requirements induce an ordering on the set of admissible structures in terms of their complexity. The more information is required, the more complex the structure.

2.2 Analysis

The aim is to partition the space of admissible organizational structures in terms of the detail of information (no information, ordinal, or cardinal information) that is necessary and sufficient to find the assignment(s) that maximize(s) the expected profit for a given structure.

To establish this level one has to check the level of information used in deriving the necessary and sufficient conditions that characterize optimal assignments. Establishing the best assignments can be done by comparing the expected profits of a structure for all possible assignments. As was anticipated in the previous section, particularly useful are the profit comparisons that result from a pair–wise switch of agents. Such pair–wise switches play an important role in the proofs of the propositions for the following reason.

Suppose (PA) holds, where (PA) stands for "ordinal information (respectively, no information) is not sufficient to correctly assign *any* pair of agents i and j to a designated pair of nodes (σ_l, σ_m) of a given structure Σ, *irrespective* of the assignment of the remaining agents to the remaining nodes."

If (PA) holds, then ordinal information (no information) is not sufficient to establish global optimality of any assignment: one necessary condition for an assignment to be globally optimal is that a pair–wise switch starting at this assignment cannot lead to an improvement. If (PA) holds—a statement concerning all possible assignments of agent, hence including the optimal one—ordinal information (no information) is not sufficient to verify that this is the case.

Note that (PA) involves all possible assignments of agents within a given structure. It may therefore seem cumbersome to use (PA) as a means for rejecting the hypothesis that ordinal information (no information) is sufficient to establish global optimality. In practice it is not. Checking whether (PA) holds for a specific pair of nodes (σ_l, σ_m) depends predominantly on the functional form of $f^q(\Lambda \backslash \sigma_l, \sigma_m)$. Large part of the proofs therefore amount to judiciously choosing pairs of nodes such that (PA) holds. The gist of much of the proofs used to show that, say, ordinal information is not sufficient to establish global optimality can be seen from the following example.

Example (continued) Suppose $\Sigma = \sigma\mathcal{O}(\sigma_a, \sigma_{aa}, \sigma_{ar}\mathcal{P}\sigma_{arr})$, and $\phi(1) = \sigma$, $\phi(2) = \sigma_a$, $\phi(3) = \sigma_{aa}$, $\phi(4) = \sigma_{ar}$ and $\phi(5) = \sigma_{arr}$. From the example on page 8 we know that switching agents 3 and 4 gives rise to a change in probability of acceptance equal to

$$p_1^q\,(p_3^q - p_4^q)\,[p_2^q + p_5^q(1-p_2^q) - (1-p_2^q)] \tag{8}$$

where the expression in square brackets is $f^q(\Lambda \backslash \sigma_{aa}, \sigma_{ar})$. If ordinal information is to be sufficient to establish that the initial assignment is better (or worse), the expression in equation 8 should have opposite signs for projects of different qualities. Were this not the case, because, say, both the probability with which good projects and the probability with which bad projects are accepted rise, the net effect on expected profit would depend on the exact characteristics of the agents (in combination with α, X and Y), i.e., on cardinal information.

Note that

$$\text{sign } p_1^g = \text{sign } p_1^b \tag{9}$$

holds and that

$$\text{sign } (p_3^g - p_4^g) = -\text{sign } (p_3^b - p_4^b) \tag{10}$$

holds by assumption. Hence, for ordinal information to be sufficient to determine the better assignment

$$\text{sign } f^g(\Lambda \backslash \sigma_{aa}, \sigma_{ar}) = \text{sign } f^b(\Lambda \backslash \sigma_{aa}, \sigma_{ar}) \tag{11}$$

must hold. If one only possesses ordinal information about the agents, and one does not know the exact probabilities of acceptance, these probabilities could be arbitrarily close to 1 for good projects and arbitrarily close to 0 for bad projects. However, in that case $f^g(\Lambda \backslash \sigma_{aa}, \sigma_{ar}) \uparrow 1$, whereas $f^b(\Lambda \backslash \sigma_{aa}, \sigma_{ar}) \downarrow -1$, implying that $f^q(\cdot)$ takes on opposite signs for argument values close to 1 and 0, respectively. Therefore, ordinal information is not enough to determine whether agent 3 or 4 should be assigned to σ_{aa} or σ_{ar}. Now observe that conditions 9 and 10 hold independent of the identity of the agents and that the line of reasoning that showed that condition 11 does not hold can be applied for all possible assignments of agents to nodes in $\Lambda \backslash \sigma_{aa}, \sigma_{ar}$. In particular, it can also be applied to the globally optimal assignment. This shows that ordinal information is not sufficient to characterise the globally optimal assignment in $\Sigma = \sigma \mathcal{HO}(\sigma_a, \sigma_{aa}, \sigma_{ar} \mathcal{P} \sigma_{arr})$. ◇

This example demonstrates the usefulness of pair–wise switches. I am now ready to give two lemmas that formally state the relevance of pair–wise switches.

Lemma 1 *Consider a structure Σ, an assignment of agents i and j to nodes σ_l and σ_m, respectively, and any assignment of agents to the remaining nodes. Switch the position of agents i and j and consider the resulting difference in expected profit $\Delta E(\Pi; \sigma_l, \sigma_m)$. If the condition*

$$f^q(\Lambda \backslash \sigma_l, \sigma_m) \equiv 0 \tag{12}$$

for $q \in \{g, b\}$ does not hold for any pair of assignment of agents, then "no information" is not sufficient to correctly allocate agents to σ_l and σ_m.

The proof is simple. If "no information" about the quality of the heterogeneous agents is to be sufficient to find the best assignment, the expected profit should not depend on the assignment of agents. In particular, no pair–wise switch should lead to a change in organizational performance: $\Delta E(\Pi; \sigma_l, \sigma_m) = 0$ for all possible assignments and all nodes. Given that $p_i^q - p_j^q \neq 0$ for all $i \neq j$, and $p^q(\sigma \to \omega_{l,m}) > 0$ for all pairs of nodes (σ_l, σ_m), the only way to ensure $\Delta E(\Pi; \sigma_l, \sigma_m) = 0$ is by imposing the condition in Equation 12. $\square$

Lemma 2 provides the main tool for rejecting the hypothesis that ordinal information is sufficient for classes of structures.

Lemma 2 *Consider a structure Σ, an assignment of agents i and j to nodes σ_l and σ_m, respectively, and any assignment of agents to the remaining nodes. Switch the position of agents i and j and consider the resulting difference in expected profit $\Delta E(\Pi; \sigma_l, \sigma_m)$. If the condition*

$$\text{sign}[f^g(\Lambda \backslash \sigma_l, \sigma_m)] = \text{sign}[f^b(\Lambda \backslash \sigma_l, \sigma_m)] \tag{13}$$

cannot be shown to hold for any assignment of agents using ordinal information only, then ordinal information is not sufficient to establish the globally optimal assignment of agents to σ_l and σ_m.[3]

Structures for which cardinal information is *necessary* (and sufficient) are therefore characterized in a negative way, as those for which ordinal information is not sufficient. Lemma 1 can now be used to reject the hypothesis that "no information" is sufficient in hybrid or dual structures.

Proposition 1 *If Σ is hybrid or dual, "no information" is not sufficient to establish the optimal assignment.*

[3]The proof of this lemma and of all other results can be found in the appendix.

Lemma 2 is used to distinguish structures for which ordinal information is sufficient from those for which cardinal information is required. If one merely possesses ordinal information, establishing the sign of $f^q(\Lambda\backslash\sigma_j, \sigma_l)$ for both $q = b$ and $q = g$ may be problematic, and typically requires imposing restrictions on the structure Σ of which the nodes are part. The five lemmas that follow are instrumental in this respect.

Suppose the organizational designer only has ordinal information at her disposition, and she wants to find out whether a single agent accepts more good or bad projects than an organization with structure Σ. In some sense, the single agent must be less demanding than Σ. That is, the structure Σ must require acceptance by both its first agent *and* the consecutive substructure for any ordering of the agents. Lemma 3 makes this precise.

Lemma 3 *Assume Σ contains at least two nodes and is hybrid or pure and assume there exists a node σ_m such that $\sigma_m \notin \Sigma$. If Σ is not equal to*

$$\Sigma = \sigma\mathcal{H}\Sigma_a \tag{14}$$

then ordinal information is not sufficient to establish that $p^q(\sigma_m) > p^q(\Sigma)$ holds for either $q = g$ or $q = b$, irrespective of the assignment of agents to nodes.

Condition 14 ensures that Σ accepts fewer projects than its first agent. Hence, if the single agent at σ_m accepts more projects of a certain type than the first agent of Σ — a statement that can be checked using ordinal information only — then one knows that the single agent accepts more projects than Σ.

Similarly, Lemma 4 states that if one wants to show that Σ accepts more good projects (or bad ones) than a single agent, then the structure must allow for the possibility of implementation after rejection by the first agent. In some sense, Σ must be laxer than the single agent.

Lemma 4 *Assume Σ contains at least two nodes and is hybrid or pure and assume there* *exists a node* σ_m *such that* $\sigma_m \notin \Sigma$*. If* Σ *is not equal to*

$$\Sigma = \sigma \mathcal{P} \Sigma_r \qquad (15)$$

then ordinal information is not sufficient to establish that $p^q(\sigma_m) < p^q(\Sigma)$ *holds for either* $q = g$ *or* $q = b$*, irrespective of the assignment of agents to nodes.*

Condition 15 ensures that Σ accepts more projects than its first agent. Hence, if the single agent at σ_m accepts less projects of a certain type than the first agent of Σ — a statement that can be verified using ordinal information only — then one knows that the single agent accepts less projects than Σ.

Suppose one only possesses ordinal information of the agents, and suppose one wants to establish that a single agent i accepts both more good *and* bad projects (or less good *and* bad projects) than a hybrid or pure structure Σ. Lemma 5 characterizes the necessary and sufficient conditions this structure should satisfy, and the required ordering of the agents at its two first nodes σ_l and σ_m relative to the single agent such that this can be established. Firstly, the single agent should be of intermediate quality relative to the first two agents of Σ. Moreover, if one wants to establish that the single agent accepts both more good and more bad projects than Σ, this structure should require a triple check, first by σ_l, then by σ_m, and finally by the consecutive substructure, before final implementation. The intermediate quality of the single agent i ensures that i accepts more good projects than the worse agent, but also more bad projects than the better agent. The overall structure Σ accepts less projects (be they good or bad) than the worst of the two first agents (*i*.e., $p^q(\Sigma) = p^q(\sigma_l)p^q(\sigma_m)p^q(\Sigma_{lma}) < \min[p^q(\sigma_l), p^q(\sigma_m)]$), and so still less than agent m. The intermediate quality of the single agent i ensures that i accepts more good projects than the worse agent, but also more bad projects than the better agent.

If instead one wants to establish that the single agent accepts less bad, but also less good projects than Σ, then implementation by Σ should still be possible after rejection by both σ_l and σ_m. The overall structure Σ accepts more projects (be they good or bad) than the best of the two first agents (*i.e.*, $p^q(\Sigma) = p^q(\sigma_l) + (1 - p^q(\sigma_l))(p^q(\sigma_m) + (1 - p^q(\sigma_m))p^q(\Sigma_{lmr})) > \max[p^q(\sigma_l), p^q(\sigma_m)]$), and so still more than agent i. The intermediate quality of the single agent i ensures that i accepts less good projects than the best agent, but also less bad projects than the worst agent. To state this formally, let $\sigma_k \succ \sigma_m$ denote that the agent assigned to node σ_k is better than the one at σ_m. A related piece of notation will be useful in the proofs: let $\sigma_k \sim \sigma_m$ denote that the agents at nodes σ_m and σ_{ma} can be switched without affecting the expected payoff

Lemma 5 *Suppose* Σ *is hybrid or pure and suppose there is a node* $\sigma_k \notin \Sigma$. *The expression*

$$\text{sign}[p^g(\sigma_k) - p^g(\Sigma)] = \text{sign}[p^b(\sigma_k) - p^b(\Sigma)] \tag{16}$$

can be shown to hold using exclusively ordinal information if and only if $\exists_{\sigma_l, \sigma_m \in \Sigma} : \sigma_l \succ \sigma_k \succ \sigma_m$ *or* $\sigma_m \succ \sigma_k \succ \sigma_l$ *and* $\Sigma = \sigma_l \mathcal{H} \sigma_m \mathcal{H} \Sigma_{lma}$ *or* $\Sigma = \sigma_l \mathcal{P} \sigma_m \mathcal{P} \Sigma_{lmr}$

Lemma 6 addresses the following situation. Take any two hybrid structures Σ_l and Σ_m that require acceptance by the first node and by the successive substructure, and consider any assignment of agents to positions in these structures. Then ordinal information is not enough to establish which of the two organizations accepts both more good and more bad projects. This is because the restrictions one needs to impose on the ordering of agents of the two structures to ensure that, say, Σ_m accepts more good projects than Σ_l, are in conflict with the restrictions needed to prove that Σ_m also accepts more bad projects than Σ_l.

Lemma 6 *Assume* Σ_l *and* Σ_m *are hybrid, and that* $\Sigma_l = \sigma_l \mathcal{H} \Sigma_{la}$ *and* $\Sigma_m = \sigma_m \mathcal{H} \Sigma_{ma}$. *Then ordinal information is not sufficient to show* $\text{sign}[p^g(\Sigma_l) - p^g(\Sigma_m)] = \text{sign}[p^b(\Sigma_l) - p^b(\Sigma_m)]$.

The point of departure for the final lemma is the necessary condition stated in Lemma 2. It formalises the argument provided in the example on page page 10: if $\text{sign}[f^g(\Lambda \backslash \sigma_l, \sigma_m)] = \text{sign}[f^b(\Lambda \backslash \sigma_l, \sigma_m)]$ does not hold, then ordinal information is not sufficient to correctly allocate a pair of agents to σ_l and σ_m, as one cannot guarantee an unambiguous change in net profits.

Suppose for the sake of argument that agents are no longer error–prone, *i.e.*, $p_i^g = 1$ and $p_i^b = 0$, and suppose that in this case the necessary condition is not met, or $f^g(\Lambda \backslash \sigma_l, \sigma_m)$ and $f^b(\Lambda \backslash \sigma_l, \sigma_m)$ have opposite signs. Now consider the case of agents that are only marginally error–prone. That is, agents accept virtually all good projects and reject almost all bad projects. Hence, irrespective of the assignment of agents to nodes, the values of $f^g(\Lambda \backslash \sigma_l, \sigma_m)$ and $f^b(\Lambda \backslash \sigma_l, \sigma_m)$ are just slightly different from what they are in case of no error–prone agents (because of the continuity of $f^q(\Lambda \backslash \sigma_l, \sigma_m)$). Therefore, $f^g(\Lambda \backslash \sigma_l, \sigma_m)$ and $f^b(\Lambda \backslash \sigma_l, \sigma_m)$ are still of opposite sign, and ordinal information is not sufficient to correctly position agents at nodes σ_l and σ_m.

Lemma 7 *Suppose* $f^g(\Lambda \backslash \sigma_l, \sigma_m) \to x$ *if* $p^g(\sigma_k) \to 1$ *for all* $\sigma_k \in \Lambda \backslash \sigma_l, \sigma_m$ *and* $f^b(\Lambda \backslash \sigma_l, \sigma_m) \to y$ *if* $p^b(\sigma_k) \to 0$ *for all* $\sigma_k \in \Lambda \backslash \sigma_l, \sigma_m$. *If* $\text{sign}[x] = -\text{sign}[y]$, *then ordinal information is not sufficient to determine the optimal ordering of (agents at)* σ_l *and* σ_m.

This lemma is very useful in the proof of Proposition 2, as $f^q(\Lambda \backslash \sigma_l, \sigma_m)$ can be easily evaluated for $p_i^g = 1$ and $p_i^b = 0$ for all i. Note carefully that the lemma does not state that x should equal 1 and that y should equal 0. This does not have to hold as $f^q(\Lambda \backslash \sigma_l, \sigma_m)$ is *not* the probability of acceptance of some sub–structure. I am now able to prove the second Proposition.

Proposition 2 *If* Σ *is dual, ordinal information is not sufficient to establish the optimal assignment.*

Because of Proposition 1 the search for structures for which "no information" is sufficient to correctly allocate agents can now be limited to the class of pure structures. It is easy to see that no information is required to correctly assign heterogeneous agents to positions in any pure structure. Take a pure hierarchy of n agents. Its probability of acceptance is the product of the individual probabilities of acceptance, $p_1^q p_2^q \cdots p_n^q$. As the product operator is commutative, the assignment of the agents is immaterial. The same applies to a pure polyarchy of size n. A polyarchy accepts a project if not all its members reject it. The probability of acceptance therefore equals $1-(1-p_1)(1-p_2)\cdots(1-p_n)$. Once again, the assignment is irrelevant. That is,

Proposition 3 *If a structure Σ is pure, then "no information" is sufficient to correctly allocate agents to nodes.*

This proposition in combination with Proposition 2 shows that the search for structures for which ordinal information is sufficient can be limited to the class of hybrid structures. Proposition 4 establishes that ordinal information is sufficient for all hybrid structures.

Proposition 4 *If a structure Σ is hybrid, then ordinal information is necessary and sufficient to correctly allocate agents to nodes.*

The four Propositions 1–4 establish the main result of this paper:

Theorem *Consider any decision procedure (Σ, ϕ) satisfying Definition 1, and any ordered set of agents I. Let ϕ^* be a globally optimal assignment. "No information" is sufficient to find ϕ^* if and only if Σ is pure. Ordinal information is necessary and sufficient if and only if Σ is hybrid. Cardinal information is necessary (and sufficient) as soon as Σ contains at least one dual connection.*

The theorem shows how to classify decision structures in terms of the minimum level of information that is sufficient to ensure the structures operate optimally. The classification is based on progressively finer differences between agents (no differences, relative differences, absolute differences) that have to be discerned to establish the optimal assignment. These differences therefore form the basis of the measure of complexity introduced in this paper. Note that the complexity of the decision structure depends on the type of links between successive decision makers.

It is easy to extent this type of analysis to other structures. One structure that was not discussed, because it does not fall in the class of admissible structures, has a centre collecting the votes cast simultaneously by individual decision makers. The centre in turn decides on the basis of these votes whether the project should be implemented or not. As the quality of the votes cast differ, the optimal decision rule in this case must put weights on individual votes. Optimal weights require cardinal information.[4] Such a decision structure falls therefore in the class of most complex structures.

3 Discussion

In this paper, I have introduced a measure of the complexity of decision problems based on the degree of detail of information that is required to characterize the set of optimal solutions. It was shown how such a measure can be applied in the realm of matching and group decision problems.

Although this notion of complexity is interesting in itself, it becomes the more so when considered in conjunction with other dimensions along which to compare matching or group decision problems. For example, if the organizational designer lacks the level of detail that is required to assure optimality of an assignment of employees within a given decision structure, various routes are open to her: she can invest in extra monitoring to obtain the missing information; she may simplify the structure in line with the information she does have; or she may accept a possible erroneous assignment. Whatever is the case, complexity of a structure should be compared to its expected performance, and to its robustness. By the latter I mean the degree to which the performance of the decision structure depends on the correctness of the optimal assignment. A fourth aspect is the degree of sensitivity to changes in the characteristics of the project (α, X and Y). Because such an analysis is well beyond the scope of this paper and because of its difficulty in general terms, I have started to address these trade–offs in a situation with only three agents in Visser (2002).

When dealing with complexity, the economics literature has focused on the complexity

[4] Ben–Yashar and Nitzan (1997) characterize the optimal weights.

of implementing a strategy in games. In a repeated game setting Rubinstein (1986) and Abreu and Rubinstein (1988) measure complexity by the extent to which a player's strategy depends on history. In particular, as strategies are represented by automata, the complexity of a strategy is measured by the number of states of the automaton. These papers focus on the effect complexity considerations have on the possible Nash equilibrium strategies and outcomes of repeated games when players care both about payoffs and complexity. This has been called 'state complexity', to distinguish it from 'response complexity', by Chatterjee and Sabourian (2000). By the latter they mean the number of possible replies to the set of possible histories. They combine both notions of complexity in their analysis of a repeated n-person bargaining game, and show how such considerations reduce the multiplicity of possible equilibrium behaviour.

Another approach to complexity can be found in Rubinstein (1993). As it studies computational complexity, it is more akin to the approach advocated in the present paper. He models consumers who differ in terms of the type of information partition they consider when devising their optimal responses to a price offer by a monopolist. These differences in the type of partition, as captured by the number of threshold values it may contain, reflect differences in computational abilities. This is very similar to an organizational designer who devises the decision structure that is best given the informational limitations she faces.

Appendix: Proofs

Proof of Lemma 2 For the moment limit attention to the substructure $\Sigma(\omega_{l,m})$. Suppose one wants to determine whether $\Delta E(\Pi; \sigma_l, \sigma_m) > 0$. If

$$\Delta E(\Pi; \sigma_l, \sigma_m) = \alpha X \psi^g(\sigma_l, \sigma_m) - (1-\alpha) Y \psi^b(\sigma_l, \sigma_m) > 0 \qquad \text{(A.1)}$$

then the agents located at node σ_l and at σ_m are well positioned, while if the difference is negative, the agents should switch nodes. Equation A.1 is equivalent to

$$\frac{(p^g(\sigma_l) - p^g(\sigma_m))}{(p^b(\sigma_l) - p^b(\sigma_m))} \frac{f^g(\Lambda \backslash \sigma_l, \sigma_m)}{f^b(\Lambda \backslash \sigma_l, \sigma_m)} > \frac{1-\alpha}{\alpha} \frac{Y}{X} \quad \text{for } \psi^b(\sigma_l, \sigma_m) > 0 \tag{A.2}$$

$$\frac{(p^g(\sigma_l) - p^g(\sigma_m))}{(p^b(\sigma_l) - p^b(\sigma_m))} \frac{f^g(\Lambda \backslash \sigma_l, \sigma_m)}{f^b(\Lambda \backslash \sigma_l, \sigma_m)} < \frac{1-\alpha}{\alpha} \frac{Y}{X} \quad \text{for } \psi^b(\sigma_l, \sigma_m) < 0 \tag{A.3}$$

I show that if one is in possession of ordinal information only, Equation A.2 cannot be shown to hold, while Equation A.3 can be shown to hold only if the condition in Equation 13 is satisfied.

Given the limitation to ordinal information, the values of the constituent parts on the left hand side of Equations A.2 and A.3 are not known. Consider first Equation A.2. Since both the right hand side and the denominator of the left hand sights are larger than zero, $(p^g(\sigma_l) - p^g(\sigma_m))f^g(\Lambda \backslash \sigma_l, \sigma_m) > 0$ is a necessary condition for Equation A.2 to be verified.

The intuition behind the proof is to fix a certain allocation of agents to nodes, and to vary the values of the agents' characteristics, while respecting the ordering of the agents. Remember that the word "ordering" refers to the ordering of the screening qualities of agents at various nodes ("agent located at node σ_l is better than agent located at σ_m"). Suppose a certain allocation ϕ applies, inducing an ordering of agents over organizational nodes, and conduct the following mental experiment. Keep the value of $(p^b(\sigma_l) - p^b(\sigma_m))$ and of $f^b(\Lambda \backslash \sigma_l, \sigma_m)$ fixed, while reducing the difference between $p^g(\sigma_l)$ and $p^g(\sigma_m)$, without violating the ordering of σ_l and σ_m. Since $\Sigma(\sigma_l, \sigma_m)$ contains just a finite number of nodes, $0 \leq |f^g(\Lambda \backslash \sigma_l, \sigma_m)| < M$. Therefore,

$$\frac{(p^g(\sigma_l) - p^g(\sigma_m))}{(p^b(\sigma_l) - p^b(\sigma_m))} \frac{f^g(\Lambda \backslash \sigma_l, \sigma_m)}{f^b(\Lambda \backslash \sigma_l, \sigma_m)} \to 0 \quad \text{for } p^g(\sigma_l) \to p^g(\sigma_m) \tag{A.4}$$

Similarily, it can be shown that

$$\frac{(p^g(\sigma_l) - p^g(\sigma_m))}{(p^b(\sigma_l) - p^b(\sigma_m))} \frac{f^g(\Lambda \backslash \sigma_l, \sigma_m)}{f^b(\Lambda \backslash \sigma_l, \sigma_m)} \to \infty \quad \text{for } p^b(\sigma_l) \to p^b(\sigma_m) \tag{A.5}$$

without violating the ordering of the agents. That is, one and the same ordering of agents can give rise to any positive number. Hence, ordinal information is not enough to show that Equation A.2 holds. For the same reason, ordinal information is not sufficient in case of

Equation A.3 if $(p^g(\sigma_l) - p^g(\sigma_l))f^g(\Lambda\sigma_l, \sigma_m) < 0$. Indeed, the only possibility when ordinal information may be sufficient is when

$$\frac{(p^g(\sigma_l) - p^g(\sigma_m))}{(p^b(\sigma_l) - p^b(\sigma_m))} \frac{f^g(\Lambda \backslash \sigma_l, \sigma_m)}{f^b(\Lambda \backslash \sigma_l, \sigma_m)} < 0 \tag{A.6}$$

for $(p^b(\sigma_l) - p^b(\sigma_m))f^b(\Lambda \backslash \sigma_l, \sigma_m) < 0$. Because of Assumption 1, the expression

$$\frac{p^g(\sigma_l) - p^g(\sigma_l)}{p^b(\sigma_l) - p^b(\sigma_l)} < 0$$

holds for every pair of agents and therefore, in the light of Equation A.6,

$$\text{sign}[f^g(\Lambda \backslash \sigma_l, \sigma_m)] = \text{sign}[f^b(\Lambda \backslash \sigma_l, \sigma_m)] \tag{A.7}$$

is a necessary condition for ordinal information to be sufficient. This is condition 13.

Extending the analysis from the substructure $\Sigma(\omega_{l,m})$ to the full structure Σ adds a factor

$$\frac{p^g(\sigma \to \omega(\sigma_l, \sigma_m))}{p^b(\sigma \to \omega(\sigma_l, \sigma_m))} \tag{A.8}$$

to the left hand side of Equations A.2 and A.3, the value of which is not known as one possesses ordinal information only. Its introduction can therefore not alter the conclusion reached on the basis of the substructure as to whether ordinal information is sufficient or not to correctly allocate a pair of agents to σ_l and σ_m. □

Proof of Proposition 1 Distinguish the case where Σ is (*i*) hybrid and (*ii*) dual. Case (*i*): if one starts moving into the structure starting at the root at some point one encounters a substructure equal to either (*ia*) $\sigma_l \mathcal{H} \sigma_{la} \mathcal{P} \Sigma_{lar}$ or (*ib*) $\sigma_l \mathcal{P} \sigma_{lr} \mathcal{H} \Sigma_{lra}$. For notational purposes, it will be convenient to treat σ_l as the root σ. The original root will be denoted by σ_0. Rewrite therefore (*ia*) and (*ib*) as $\sigma \mathcal{H} \sigma_a \mathcal{P} \Sigma_{ar}$ and $\sigma \mathcal{P} \sigma_r \mathcal{H} \Sigma_{ra}$, respectively. In (*ia*), consider any pair of agents i and j initially assigned to σ and σ_a, respectively. Switch these agents. The resulting difference in probability of acceptance equals $(p_i - p_j)p(\Sigma_{ar})p(\sigma_0 \to \omega_{\sigma,a})$, such that $f(\Lambda \backslash \sigma, \sigma_a) = p(\Sigma_{ar})$. As $p(\Sigma_{ar}) \neq 0$ (if $p(\Sigma_{ar}) = 0$ the structure would not be hybrid), one can now apply Lemma 1: "no information" is not sufficient to find the best allocation. A similar line of reasoning shows that in case (*ib*) "no information" is not sufficient.

In case (ii), at some point one encounters the substructure $\mathcal{O}(\sigma_l, \Sigma_{la}, \Sigma_{lr})$, with both Σ_{la} and Σ_{lr} either simple or hybrid. One can always find such a substructure as Σ is dual. Note that either Σ_{la} or Σ_{lr} or both may consist of a single node. Apply the same transformation in notation and write $\Sigma = \mathcal{O}(\sigma, \Sigma_a, \Sigma_r)$. Consider any pair of agents i and j initially assigned to σ and σ_a, respectively. Switch these agents. The ensuing difference in probability of acceptance equals $(p_i - p_j)(p(\Sigma_{ar}) - p(\Sigma_r))p(\sigma_0 \to \omega_{\sigma,a})$, where Σ_{ar} is possibly empty (*i.e.*, when $\Sigma_a = \sigma_a \mathcal{H} \Sigma_{aa}$ or $\Sigma_a = \sigma_a$, in which case $p(\Sigma_{ar}) = 0$), but, by definition, Σ_r contains at least one node, σ_r. Hence, $f(\Lambda \backslash \sigma, \sigma_a) = p(\Sigma_{ar}) - p(\Sigma_r)$. For "no information" to be sufficient, $f(\Lambda \backslash \sigma, \sigma_a) \equiv 0$ must hold, implying at least that Σ_r should be an empty structure. This violates the requirement of Σ being dual. That is, "no information" is not sufficient as soon as a structure contains one dual connection. □

Proof of Lemma 3 The intuition of the proof can be obtained by limiting Σ to two nodes. The formal proof that follows establishes that what holds in this simple case carries over to general structures satisfying the conditions of the lemma. Take $\Sigma_1 = \sigma \mathcal{P} \sigma_r$ and $\Sigma_2 = \sigma \mathcal{H} \sigma_a$. With ordinal information only it is impossible to show that the inequality $p^q(\sigma_m) > p^q(\Sigma_1) = p^q(\sigma) + (1 - p^q(\sigma))p^q(\sigma_r)$ holds: even if $p^q(\sigma_m) > \max(p^q(\sigma), p^q(\sigma_r))$, it can still be that $p^q(\sigma_m) < p^q(\Sigma_1)$. Knowledge of the ordering of the agents is not enough to make this decidable. However, that the inequality $p^q(\sigma_m) > p^q(\Sigma_2) = p^q(\sigma)p^q(\sigma_a)$ holds, can be shown with ordinal information when specific restrictions on the ordering of (agents at) $\{\sigma_m, \sigma, \sigma_a\}$ are satisfied: if $p^q(\sigma_m) > p^q(\sigma_k)$ or if $p^q(\sigma_m) > p^q(\sigma_{ka})$ holds, then clearly $p^q(\sigma_m) > p^q(\Sigma_2)$. Therefore, if Σ is not equal to $\Sigma = \sigma_k \mathcal{H} \sigma_{ka}$, then ordinal information cannot be sufficient to establish that $p^q(\sigma_m) > p^q(\Sigma)$. That is, the lemma is correct in case of two nodes.

Formally, in the general case one wants to show $p^g(\sigma_m) > p^g(\Sigma)$ using ordinal information only (the same line of reasoning applies for $q = b$). Then, either (i) $\Sigma = \sigma \mathcal{P} \Sigma_r$ or (ii) $\Sigma = \sigma \mathcal{H} \Sigma_a$ as Lemma 3 supposes that Σ is pure or hybrid. In case (i), $p^g(\sigma_m) > p^g(\Sigma)$ amounts to $p^g(\sigma_m) > p^g(\sigma_l) + (1 - p^g(\sigma_l))p^g(\Sigma_{lr})$. This is equivalent to $p^g(\sigma_m) - p^g(\sigma_l) >$

$(1 - p^g(\sigma_l))p^g(\Sigma_{lr})$. Since the right hand side of this inequality is positive, one needs at least that the left hand sight is also positive. That is, $\sigma_m \succ \sigma_l$ is a necessary condition. However, with ordinal information only this condition does not exclude for any ordering and any Σ_r the possibility that $p^g(\sigma_m) - p^g(\sigma_l)$ almost vanishes, while the right hand side $(1 - p^g(\sigma_l))p^g(\Sigma_{lr})$ is bounded away from zero for every finite structure. Take, *e.g.*, the case where the probability of acceptance of good projects is just somewhat larger than one half for every agent. Therefore, ordinal information is not enough in this case.

In case (ii), where $\Sigma = \sigma\mathcal{H}\Sigma_a$, it is straightforward to find a sufficient condition which ensures that $p^g(\sigma_m) > p^g(\Sigma)$ holds. Note that $p^g(\Sigma)$ equals $p^g(\sigma)p^g(\Sigma_a)$. Clearly, if $p^g(\sigma_m) > p^g(\sigma)$ holds, then $p^g(\sigma_m) > p^g(\sigma)p^g(\Sigma_a)$ holds, whatever the ordering of the nodes in Σ_a. That is, ordinal information can be sufficient in this case. □

Proof of Lemma 4 The proof used for Lemma 3 also applies, *mutatis mutandis*, to this lemma. □

Proof of Lemma 5. Once again, intuition can be provided by showing the two nodes case. Suppose $\sigma_l \succ \sigma_k \succ \sigma_m$, such that $p^g(\sigma_l) > p^g(\sigma_k) > p^g(\sigma_m)$ and $p^b(\sigma_m) > p^b(\sigma_k) > p^b(\sigma_l)$ hold. For $\Sigma = \sigma_l\mathcal{H}\sigma_m$, a hierarchy, this amounts to $p^g(\sigma_k) > p^g(\sigma_l)p^g(\sigma_m)$ and $p^b(\sigma_k) > p^b(\sigma_l)p^b(\sigma_m)$, whereas for a polyarchy, $\Sigma = \sigma_l\mathcal{P}\sigma_m$, $p^g(\sigma_k) < p^g(\sigma_l) + (1 - p^g(\sigma_l))p^g(\sigma_m)$ and $p^b(\sigma_k) < p^b(\sigma_l) + (1 - p^b(\sigma_l))p^b(\sigma_m)$ hold. In either case, $\text{sign}[p^g(\sigma_k) - p^g(\Sigma)] = \text{sign}[p^b(\sigma_k) - p^b(\Sigma)]$. And now formally.

(Sufficiency). Take $\Sigma = \sigma_l\mathcal{H}\sigma_m\mathcal{H}\Sigma_{lma}$, with Σ_{lma} pure or hybrid (as Σ should be pure or hybrid) and $\sigma_l \succ \sigma_k \succ \sigma_m$ (the proof carries over, *mutatis mutandis*, to the case of $\sigma_m \succ \sigma_k \succ \sigma_l$). From $\sigma_k \succ \sigma_m$, it follows that $p^g(\sigma_k) > p^g(\sigma_m) > p^g(\sigma_m)p^g(\sigma_l)p^g(\Sigma_{lma})$. From $\sigma_l \succ \sigma_k$, it follows that $p^b(\sigma_k) > p^b(\sigma_l) > p^b(\sigma_m)p^b(\sigma_l)p^b(\Sigma_{lma})$. Therefore, $\text{sign}[p^g(\sigma_k) - p^g(\Sigma)] = \text{sign}[p^b(\sigma_k) - p^b(\Sigma)] = +$. The same line of reasoning shows that $\Sigma = \sigma_l\mathcal{P}\sigma_m\mathcal{P}\Sigma_{lmr}$, with Σ_{lmr} pure or hybrid and $\sigma_l \succ \sigma_k \succ \sigma_m$ is sufficient.

(Necessity). Necessity can be shown by establishing that ordinal(i) when $\sigma_l \succ \sigma_k \succ \sigma_m$ or $\sigma_m \succ \sigma_k \succ \sigma_l$ do not hold and (ii) when structures are complementary to $\Sigma = \sigma_l \mathcal{H} \sigma_m \mathcal{H} \Sigma_{lma}$ and $\Sigma = \sigma_l \mathcal{P} \sigma_m \mathcal{P} \Sigma_{lmr}$. First case ($i$). If the agent located at σ_k is better (or worse) than both agents located at σ_m and σ_l, sign$[p^q(\sigma_k) - p^q(\Sigma)]$ cannot be determined using ordinal information only. Now case (ii). Given that Lemma 5 limits discussion to pure and hybrid structures, the complementary structures are (ii-a) $\Sigma = \sigma_l \mathcal{P} \sigma_m \mathcal{H} \Sigma_{lma}$ and (ii-b) $\Sigma = \sigma_l \mathcal{H} \sigma_m \mathcal{P} \Sigma_{lmr}$. In case of ($ii$-$a$), Lemma 3 excludes the possibility of proving $p^g(\sigma_k) > p^g(\Sigma)$. Therefore, one can only attempt to show that $p^g(\sigma_k) < p^g(\Sigma)$ holds, *i.e.*, $p^g(\sigma_k) < p^g(\sigma_l) + (1 - p^g(\sigma_l))p^g(\sigma_m)p^g(\Sigma_{lma})$. This requires $p^g(\sigma_k) < p^g(\sigma_l)$ if ordinal information is to be sufficient. Similarly, for $p^b(\sigma_k) < p^b(\Sigma)$ to be shown to hold using ordinal information only, $p^b(\sigma_k) < p^b(\sigma_l)$ is a necessary condition. These two conditions are in conflict with each other. Therefore, ordinal information is not sufficient. The same line of reasoning shows that ordinal information is not sufficient in case of (ii-b). □

Proof of Lemma 6 Intuition for the proof can be obtained by limiting attention to the case of two nodes: both $p^g_l p^g_{la} > p^g_m p^g_{ma}$ and $p^b_l p^b_{la} > p^b_m p^b_{ma}$ should be shown to hold using ordinal information only. Necessary conditions are either $p^g_l > p^g_m$ and $p^g_{la} > p^g_{ma}$, or $p^g_l > p^g_{ma}$ and $p^g_{la} > p^g_m$. Take the first set of conditions. They amount to $\sigma_l \succ \sigma_m$ and $\sigma_{la} \succ \sigma_{ma}$. This implies for the bad projects that $p^b_l < p^b_m$ and $p^b_{la} < p^b_{ma}$ hold, and therefore $p^b_l p^b_{la} < p^b_m p^b_{ma}$, which is the wrong sign. The same holds for the second set of conditions. This proves the lemma in the two nodes case.

For general structures this amounts to analysing the case sign$[p^g(\Sigma_l) - p^g(\Sigma_l)] =$ sign$[p^b(\Sigma_l) - p^b(\Sigma_l)] = +$ (This implies no limitation as one can freely interchange the structures Σ_l and Σ_m). Since $\Sigma_l = \sigma_l \mathcal{H} \Sigma_{la}$ and $\Sigma_m = \sigma_m \mathcal{H} \Sigma_{ma}$, $p^q(\Sigma_l) > p^q(\Sigma_l)$ equals $p^q(\sigma_l)p^q(\Sigma_{la}) > p^q(\sigma_m)p^q(\Sigma_{ma})$ for $q = g, b$. Necessary conditions for this inequality to be satisfied for $q = g$ and $q = b$ simultaneously using ordinal information only are

$$p^g(\sigma_l) > p^g(\sigma_m) \ \wedge \ p^g(\Sigma_{la}) > p^g(\Sigma_{ma}) \ \wedge \ p^b(\sigma_l) > p^b(\Sigma_{ma}) \ \wedge \ p^b(\Sigma_{la}) > p^b(\sigma_m) \qquad \text{(A.9)}$$

or

$$p^b(\sigma_l) > p^b(\sigma_m) \ \wedge \ p^b(\Sigma_{la}) > p^b(\Sigma_{ma}) \ \wedge \ p^g(\sigma_l) > p^g(\Sigma_{ma}) \ \wedge \ p^g(\Sigma_{la}) > p^g(\sigma_m) \qquad \text{(A.10)}$$

First the conditions in A.9. It follows from Lemma 4 that if Σ_{ma} is not equal to $\Sigma_{ma} = \sigma_{ma}\mathcal{H}\Sigma_{maa}$ then ordinal information is not sufficient to verify $p^b(\sigma_l) > p^b(\Sigma_{ma})$. Hence, $p^b(\sigma_l) > p^b(\sigma_{ma})p^b(\Sigma_{maa})$ should hold and $\sigma_m \sim \sigma_{ma}$. Moreover, since $p^g(\sigma_l) > p^g(\sigma_m)$ is a necessary condition, $\sigma_l \succ \sigma_m$ is a necessary condition. Therefore $\sigma_l \succ \sigma_m \sim \sigma_{ma}$ is a necessary condition, which implies that $p^b(\sigma_{ma}) > p^b(\sigma_l)$ is a necessary condition. The latter implication together with $p^b(\sigma_l) > p^b(\sigma_{ma})p^b(\Sigma_{maa})$ shows that $p^b(\sigma_l) > p^b(\Sigma_{maa})$ is a necessary condition. That is, $\Sigma_{maa} = \sigma_{maa}\mathcal{H}\Sigma_{maaa}$ is a necessary condition and one enters an infinite regress. This proves that ordinal information is insufficient, since the structures are finite. In case of condition (A.10) one enters an infinite regress for the same reason. □

Proof of Lemma 7 Since the function $f(\Lambda\backslash\sigma_l, \sigma_m)$ is continuous in its arguments, *any* ordering of the nodes $\sigma_k \in \Lambda\backslash\sigma_l, \sigma_m$, with values $p^g(\sigma_k)$ sufficiently close to 1 satisfies $\text{sign}[f^g(\Lambda\backslash\sigma_l, \sigma_m)] = \text{sign}[x]$. Similarily, *any* ordering of the nodes $\sigma_k \in \Lambda\backslash\sigma_l, \sigma_m$, with values $p^b(\sigma_k)$ sufficiently close to 0 satisfies $\text{sign}[f^b(\Lambda\backslash\sigma_l, \sigma_m)] = \text{sign}[y]$.

Since by assumption, $\text{sign}[x] = -\text{sign}[y]$, this implies that whatever the ordering of the nodes of $\Lambda\backslash\sigma_l, \sigma_m$, $\text{sign}[f^g(\Lambda\backslash\sigma_l, \sigma_m)] = -\text{sign}[f^b(\Lambda\backslash\sigma_l, \sigma_m)]$ is possible. Then, by Lemma 2, ordinal information is not sufficient. □

Proof of Proposition 2 Find the substructure $\mathcal{O}(\sigma_l, \Sigma_{la}, \Sigma_{lr})$, with both Σ_{la} and Σ_{lr} either pure or hybrid. To simplify notation, let σ_0 denote the original root, and let σ stand for σ_l. Hence, consider $\mathcal{O}(\sigma, \Sigma_a, \Sigma_r)$. Now distinguish the four mutually exclusive and exhaustive possibilities: (i) $\mathcal{O}(\sigma, \sigma_a\mathcal{H}\Sigma_{aa}, \sigma_r\mathcal{P}\Sigma_{rr})$, (ii) $\mathcal{O}(\sigma, \sigma_a\mathcal{P}\Sigma_{ar}, \sigma_r\mathcal{P}\Sigma_{rr})$, (iii) $\mathcal{O}(\sigma, \sigma_a\mathcal{H}\Sigma_{aa}, \sigma_r\mathcal{H}\Sigma_{ra})$ and (iv) $\mathcal{O}(\sigma, \sigma_a\mathcal{P}\Sigma_{ar}, \sigma_r\mathcal{H}\Sigma_{ra})$.

(i) Consider any pair of agents i and j, initially assigned to nodes σ_a and σ_r, respectively. Switch these agents. The resulting change in the probability of acceptance equals

$(p_i - p_j) f(\Lambda \backslash \sigma_a, \sigma_r) p(\sigma_0 \to \omega_{a,r})$, where

$$f(\Lambda \backslash \sigma_a, \sigma_r) = p(\sigma) p(\Sigma_{aa}) + (1 - p(\sigma)) p(\Sigma_{rr}) - (1 - p(\sigma)) \tag{A.11}$$

If $p(\sigma_m) \to 1$ for all $\sigma_m \in \Lambda \backslash \sigma_a, \sigma_r$, then $f(\Lambda \backslash \sigma_a, \sigma_r) \to 1$, while if $p(\sigma_m) \to 0$ for all σ_m, then $f(\Lambda \backslash \sigma_a, \sigma_r) \to -1$. Then, by Lemma 7, ordinal information is not sufficient to determine the optimal ordering of agents to σ_a and σ_r.

(*ii*) Consider any pair of agents i and j, initially assigned to nodes σ_a and σ_r, respectively. Switch these agents. The resulting change in the probability of acceptance equals $(p_i - p_j) f(\Lambda \backslash \sigma_a, \sigma_r) p(\sigma_0 \to \omega_{a,r})$, where

$$f(\Lambda \backslash \sigma_a, \sigma_r) = p(\sigma)(1 - p(\Sigma_{ar})) - (1 - p(\sigma))(1 - p(\Sigma_{rr})) \tag{A.12}$$

If $p(\sigma_m) \to 0$ for all $\sigma_m \in \Lambda \backslash \sigma_a, \sigma_r$, then $f(\Lambda \backslash \sigma_a, \sigma_r) \to -1$ and therefore if

$$f^q(\Lambda \backslash \sigma_a, \sigma_r) < 0 \tag{A.13}$$

is not met for $q \in \{g, b\}$, ordinal information is not sufficient by Lemma 7. For $q = g$, this necessary condition can be rewritten as

$$\frac{p^g(\sigma)}{1 - p^g(\sigma)} \frac{1 - p^g(\Sigma_{ar})}{1 - p^g(\Sigma_{rr})} < 1 \tag{A.14}$$

Given that information is ordinal, it is not known whether $p^g(\sigma)$ is larger or smaller than 1/2. If one can show that for $p^g(\sigma) > 1/2$ ordinal information is not sufficient to verify Equation A.14, then ordinal information is not enough to find the optimal allocation for the class of structures covered by case (*ii*). For $p^g(\sigma) > 1/2$, the first fraction on the left–hand side of the condition is larger than one. The second fraction should therefore be "sufficiently small" to ensure the product of the two remains below one. Ordinal information is not sufficient to decide whether this is the case. Cardinal information is required.

(*iii*) Consider any pair of agents i and j, initially assigned to nodes σ_a and σ_r, respectively. Switch these agents. The resulting change in the probability of acceptance equals $(p_i -$

$p_j)f(\Lambda\backslash\sigma_a,\sigma_r)p(\sigma_0 \to \omega_{a,r})$, where

$$f^q(\Lambda\backslash\sigma_a,\sigma_r) = p(\sigma)p(\Sigma_{aa}) - (1-p(\sigma))p(\Sigma_{ra}) \tag{A.15}$$

If $p(\sigma_m) \to 1$ for all $\sigma_m \in \Lambda\backslash\sigma_a,\sigma_r$, then $f(\Lambda\backslash\sigma_a,\sigma_r) \to 1$, and therefore, from Lemma 7 it follows that if

$$f^q(\Lambda\backslash\sigma_a,\sigma_r) > 0 \tag{A.16}$$

is not met for $q \in \{g,b\}$, ordinal information is not sufficient. In particular, for $q = b$ this amounts to

$$\frac{p^b(\sigma)}{1-p^b(\sigma)}\frac{p^b(\Sigma_{aa})}{p^b(\Sigma_{ra})} > 1 \tag{A.17}$$

Exactly the same line of reasoning as in case (ii), but now with $p^b < 1/2$ shows that ordinal information is not sufficient to verify this condition.

(iv) Consider any pair of agents i and j, initially assigned to nodes σ and σ_a, respectively. Switch these agents. The resulting change in the probability of acceptance equals $(p_i - p_j)f(\Lambda\backslash\sigma,\sigma_a)p(\sigma_0 \to \omega_{\sigma,a})$, where

$$f(\Lambda\backslash\sigma,\sigma_a) = p(\Sigma_{ar}) - p(\sigma_r)p(\Sigma_{ra}) \tag{A.18}$$

It follows from Lemma 6 that if Σ_{ar} is not equal to $\sigma_{ar}\mathcal{P}\Sigma_{arr}$ ordinal information cannot be sufficient to decide on the sign of $p^q(\Sigma_{ar}) - p^q(\sigma_r)p^q(\Sigma_{ra})$ for $q \in \{g,b\}$. Impose therefore $\Sigma_{ar} = \sigma_{ar}\mathcal{P}\Sigma_{arr}$. From this it follows that the only ordering of σ and σ_a that can potentially be shown to hold with ordinal information only is $\sigma \succ \sigma_a$[5].

Consider any pair of agents i and j, initially assigned to nodes σ_a and σ_r, respectively. Switch these agents. The resulting change in the probability of acceptance equals $(p_i -$

[5] This is true for the following reason. Note that $p^q(\Sigma_{ar}) > p^q(\sigma_{ar})$ and $p^q(\Sigma_r) < p^q(\sigma_r)$. If $\sigma_r \succ \sigma_{ar}$ then $\text{sign}[p^q(\Sigma_{ar}) - p^q(\Sigma_r)]$ cannot be determined with ordinal information for either $q = g$ or $q = b$. If $\sigma_{ar} \succ \sigma_r$, then $\text{sign}[p^q(\Sigma_{ar}) - p^q(\Sigma_r)] = +$ for $q = g$ and additional restrictions on the ordering of agents at nodes in Σ_{ar} and Σ_r may make ordinal information sufficient to show $\text{sign}[p^b(\Sigma_{ar}) - p^b(\Sigma_r)] = +$. That is, $\sigma \succ \sigma_a$ is the only ordering that can potentially be shown using exclusively ordinal information

$p_j)f(\Lambda\backslash\sigma_a,\sigma_r)p(\sigma_0 \to \omega_{a,r})$, where

$$f(\Lambda\backslash\sigma_a,\sigma_r) = p(\sigma) - (p(\sigma)p(\Sigma_{ar}) + (1-p(\sigma))p(\Sigma_{ra})) \quad \text{(A.19)}$$

Clearly, if $p(\sigma)^q <$min $(p^q(\Sigma_{ra}), p^q(\Sigma_{ar}))$ for $q = g, b$, then $f(\Lambda\backslash\sigma_a,\sigma_r) < 0$ for $q = g, b$. If $\Sigma_{ar} = \sigma_{ar}\mathcal{P}\sigma_{arr}$ or $\Sigma_{ar} = \sigma_{ar}\mathcal{P}\sigma_{arr}\mathcal{P}\Sigma_{arrr}$ and $\sigma_{ar} \succ \sigma \succ \sigma_{arr}$ or $\sigma_{ar} \succ \sigma \succ \sigma_{arr}$ do not hold, ordinal information is not sufficient to establish whether $p^q(\sigma) < p^q(\Sigma_{ar})$ holds for $q = g$ and $q = b$. This follows from Lemma 5. Now note that the necessary condition $\sigma \succ \sigma_a$ derived from Equation A.18 also applies to the relationship between σ and σ_{ar}, and between σ and σ_{arr} as $\{\sigma_a, \sigma_{ar}, \sigma_{arr}\}$ are connected by polyarchical connections. Hence, $\sigma \succ \sigma_{ar}$ and $\sigma \succ \sigma_{arr}$ are necessary conditions. However, these conditions are in conflict with either ordering of (agents at) $\{\sigma_{ar}, \sigma, \sigma_{arr}\}$ derived from Equation A.19. Therefore, ordinal information is not enough to derive the optimal ordering of these nodes. Exactly the same line of reasoning applies to the case where $p^q >$ max $(p^q(\Sigma_{ra}), p^q(\Sigma_{ar}))$ for $q = g, b$, but this time the first nodes of both Σ_{ar} and Σ_{ra} are connected by hierarchical connections . Finally the case where

$$\min(p^q(\Sigma_{ra}), p^q(\Sigma_{ar})) < p(\sigma)^q < \max(p^q(\Sigma_{ra}), p^q(\Sigma_{ar})) \quad \text{(A.20)}$$

and consider $q = g$. $p^* := p^g(\Sigma_{ra})/(1 - p^g(\Sigma_{ar}) + p^g(\Sigma_{ra}))$ solves

$$p^g(\sigma)p^g(\Sigma_{ar}) + (1 - p^g(\sigma))p^g(\Sigma_{ra}) = p^g(\sigma) \quad \text{(A.21)}$$

in $p^g(\sigma)$. Since $p^* \in$ (min$(p^q(\Sigma_{ra}), p^q(\Sigma_{ar}))$,max$(p^q(\Sigma_{ra}), p^q(\Sigma_{ar})))$ by construction, there exist a $p_1^* := p^* - \epsilon_1$ and a $p_2^* := p^* + \epsilon_2$, with $\epsilon_i > 0$ such that $p_i{}^* \in$ (min$(p^g(\Sigma_{ra}), p^g(\Sigma_{ar}))$, max$(p^g(\Sigma_{ra}), p^g(\Sigma_{ar})))$, $i = 1, 2$ and such that changing the value of $p^g(\sigma)$ from p^* to either $p_1{}^*$ or $p_2{}^*$, does not change the ordering of σ relative to the nodes in Σ_{ar} and Σ_{ra}. The change of value of $p^g(\sigma)$ from p^* to either p_1^* or p_2^* does upset the equality of Equation A.21. Therefore, any given ordering of the nodes of Σ_{ar}, Σ_{ra} and σ satisfying the condition of Equation A.20 can give rise to $f^g(\Lambda\backslash\sigma_a,\sigma_r) = 0$, $f^g(\Lambda\backslash\sigma_a,\sigma_r) < 0$, and $f^g(\Lambda\backslash\sigma_a,\sigma_r) > 0$. That is, ordinal information is not sufficient to establish the ordering of σ_a and σ_r. □

Proof of Proposition 4 As the proposition makes a statement about the entire class of hybrid structures and since structures can be recursively defined, I use mathematical induction. In the *basis step* one proves that the statement holds for the basic structures with a specific number of nodes. One then supposes that the statement holds for any structure containing at most n nodes and then proves that the statement holds for any structure containing $n+1$ nodes. This is called the *hypothesis step* or the *induction hypothesis.*

First the basis step. Consider the two simplest hybrid structures, (i) $\sigma\mathcal{P}\sigma_r\mathcal{H}\sigma_{ra}$ and (ii) $\sigma\mathcal{H}\sigma_a\mathcal{P}\sigma_{ar}$. Consider case (i) and the associated probability of acceptance function $p(\sigma) + (1 - p(\sigma))(p(\sigma_r)p(\sigma_{ra}))$. Obviously, the agents assigned to nodes σ_r and σ_{ra} can be switched without affecting the performance of the organization. Hence, $\sigma_r \sim \sigma_{ra}$. Therefore, what has to be established is the relationship between the agent at the first node and the agents at the successive nodes. Consider any pair of agents i and j, initially assigned to nodes σ and σ_r, respectively. Switch these agents. Then $f^q(\cdot) = 1 - p(\sigma_{ra})$. As $0 < p(\sigma_{ra}) < 1$, this shows that the better agent should be located at σ, and the worse agent at σ_r. Of course, the same ordering must hold for the agents located at the pair (σ, σ_{ra}). Hence, one knows that $\sigma \succ \sigma_r$, $\sigma \succ \sigma_{ra}$, and $\sigma_r \sim \sigma_{ra}$ are necessary conditions any globally optimal allocation should satisfy. Remember that the agents satisfy $1 \succ 2 \succ 3$. Hence, $\phi_1^* = (\phi_1^*(1), \phi_1^*(2), \phi_1^*(3)) = (\sigma, \sigma_r, \sigma_{ra})$ and $\phi_2^* = (\sigma, \sigma_{ra}, \sigma_a)$ are the only allocations that satisfy these necessary conditions. As they give rise to the same expected profit both are globally optimal allocations. Clearly, ordinal information is necessary and sufficient.

The same line of reasoning shows that in case (ii) the globally optimal allocation satisfies $\sigma \succ \sigma_a$, $\sigma \succ \sigma_{ar}$, and $\sigma_a \sim \sigma_{ar}$. Once again, ordinal information is necessary and sufficient. Hence, the proposition holds for the basic hybrid structures. This completes the basis step.

Now suppose the implication holds for all hybrid structures Σ, Σ_1, and Σ_2 with at most n nodes and consider (a) $\Sigma' = \sigma\mathcal{H}\Sigma$, (b) $\Sigma' = \sigma\mathcal{P}\Sigma$ and (c) $\Sigma' = \mathcal{O}(\sigma, \Sigma_1, \Sigma_2)$.

In (a), if $\Sigma' = \sigma\mathcal{H}\Sigma$ is to be hybrid then Σ should be pure or hybrid. If Σ is a hierarchy then so is Σ' and so the implication is trivially true. If Σ is a polyarchy, then Σ' is hybrid. Write $\Sigma' = \sigma\mathcal{H}\sigma_a\mathcal{P}\Sigma_{ar}$, with Σ_{ar} a polyarchy. Therefore, pair–wise switching establishes $\sigma_a \sim \sigma_{ar} \sim \cdots \sim \sigma_{ar\ldots r}$. Moreover, pair–wise switching shows that $\sigma \succ \sigma_a$ should hold for profits to be maximized and by the same token $\sigma \succ \sigma_{ar}, \ldots, \sigma_{ar\cdots r}$ etc.

As all allocations that satisfy these necessary conditions for global optimality give rise to the same expected profits, all are globally optimal. Clearly, ordinal information is necessary and sufficient.

If, on the other hand, Σ is not pure but hybrid, then, by the induction step, ordinal information is necessary and sufficient for Σ to be correctly organized. Obviously, adding node σ to Σ' does not change the level of information necessary and sufficient to correctly assign agents in Σ'. What has to be determined is the relationship between the agent to be located at σ and those to be assigned to nodes in Σ'.

Take any node $\sigma_{mj} \in \Sigma$ and consider Σ_m. Then either *(a-i)* $\Sigma_m = \sigma_m\mathcal{H}\Sigma_{ma}$ or σ_m is the final node or *(a-ii)* $\Sigma_m = \sigma_m\mathcal{P}\Sigma_{mr}$.

In *(a-i)*, if all links between σ and σ_m are hierarchical connections (or if $\sigma_m = \sigma_a$), then pair–wise switching shows that $\sigma \sim \sigma_m$. If some links are polyarchical, pair–wise switching shows that $\sigma \succ \sigma_m$ is necessary for profits to be maximized.

In *(a-ii)*, pair–wise switching establishes $\sigma \succ \sigma_m$ is necessary for profits to be maximized.

Note that the pair–wise orderings of σ and $\sigma_m \in \Sigma$ are consistent. That is, *(a-i)* and *(a-ii)* jointly establish that in *(a)* pair–wise switches lead to a set of possible allocations, all of which can easily be checked to give rise to the same expected profit. Hence, ordinal information is necessary and sufficient to characterize the globally optimal allocations. In *(b)*, with $\Sigma' = \sigma\mathcal{P}\Sigma$, the same line of reasoning applies.

Note that in case *(c)* the structure is not hybrid and therefore the implication is trivially true. $\square$

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