# Making Descriptive Use of Prospect Theory to Improve the Prescriptive Use of Expected Utility

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This paper proposes a quantitative modification of standard utility elicitation procedures, such as the probability and certainty equivalence methods, to correct for commonly observed violations of expected utility. Traditionally, decision analysis assumes expected utility not only for the prescriptive purpose of calculating optimal decisions but also for the descriptive purpose of eliciting utilities. However, descriptive violations of expected utility bias utility elicitations. That such biases are effective became clear when systematic discrepancies were found between different utility elicitation methods that, under expected utility, should have yielded identical utilities. As it is not clear how to correct for these biases without further knowledge of their size or nature, most utility elicitations still calculate utilities by means of the expected utility formula. This paper speculates on the biases and their sizes by using the quantitative assessments of probability transformation and loss aversion suggested by prospect theory. It presents quantitative corrections for the probability and certainty equivalence methods. If interactive sessions to correct for biases are not possible, then the authors propose to use the corrected utilities rather than the uncorrected ones in prescriptions of optimal decisions. In an experiment, the discrepancies between the probability and certainty equivalence methods are removed by the authors' proposal.
(*Utility Elicitation; Probability Transformation; Loss Aversion*)

This paper proposes new formulas for measuring utility in contexts of risk and uncertainty. The primary application that we have in mind concerns policy decisions, where a team of specialists has to take a decision that best represents the interests of a group of clients. Such decisions occur for instance in the health domain where cost-effectiveness studies are conducted to decide on optimal treatments for classes of patients (Gold et al. 1996, Protheroe et al. 2000, p. 1383, Stiggelbout and de Haes 2001). Our proposal concerns the final stage of the decision analysis, where the decision situation has already been structured through a decision tree and a quantitative analysis should be carried out to determine the best decision.

It is commonly assumed in decision analysis, and also in this paper, that the right normative model for decision under uncertainty is expected

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utility (the *normative assumption*). This assumption is advocated by Broome (1991, §3.1), de Finetti (1937), Edwards (1992, Ch. 10), Eeckhoudt (1996), who gives pragmatic reasons, Hammond (1988), Harsanyi (1955), Kahneman and Tversky (1979, p. 277), Savage (1954), and others. Critics include Allais (1953), Ellsberg (1961), Loomes and Sugden (1982), Machina (1982), and Schmeidler (1989). Expected utility requires that uncertainties are quantified in terms of probabilities and the values of outcomes in terms of utilities. Then, the probability-weighted average utility is taken to determine the optimal decision. Probabilities are usually inferred from statistical data and sometimes from subjective assessments by specialists. We assume that the probabilities have already been obtained and concentrate on the measurement of utility.

Utilities are usually inferred from interviews with clients. A common assumption in empirical utility measurements is that a client's answers can be analyzed through expected utility. We call this assumption the *classical elicitation assumption*. For instance, if a client expresses indifference between \$35K (K = 1,000) and a 50-50 gamble yielding either \$100K or \$0, then the classical elicitation assumption entails that the utility of \$35K is the midpoint between the utilities of \$100K and \$0. The classical elicitation assumption is essentially descriptive because it concerns observed behavior, and it is logically independent of the normative assumption. Problems can arise if the client deviates from expected utility. Then the classical elicitation assumption may yield biased utilities and even contradictions if inconsistencies are contained in the data (Richardson 1994, pp. 7–10).

Contradictions in traditional utility measurements have been discovered indeed. Karmarkar (1978) and McCord and de Neufville (1986) pointed out that the utility function elicited through the certainty equivalent method depends on the probabilities used. Hershey and Schoemaker (1985) observed systematic discrepancies between the certainty equivalent method and the probability equivalent method. That methods comparing riskless to risky options are prone to distortions has been suggested before (Davidson et al. 1957, Officer and Halter 1968, p. 259). All of these observations violate the classical elicitation assumption.

The problems caused by choice inconsistencies can be mitigated through interactive sessions, where the client is asked to reconsider inconsistent choices. As suggested by some papers on the constructive view of preference (Edwards and Elwyn 1999, Payne et al. 1999, Slovic 1995) and in line with Plott's (1996) discovered preference hypothesis, we recommend such a use of interactive sessions whenever possible. In many applications, however, interactive sessions are not possible due to practical limitations (Fischhoff 1991, p. 844, Hershey and Baron 1987, p. 210). Interactive sessions are expensive and time-consuming, and require not only sophisticated interviewers but also sophisticated clients. In the health domain, for instance, utilities often have to be elicited from the general public by hired interviewers (Gold et al. 1996). Then interactive sessions are usually impossible to implement. Our proposed new formulas are primarily developed for such situations, where the measurement has to be "quick and dirty." Another difficulty with interactively elicited utilities lies in their interpretation. These quantities have not been measured objectively, as is common in descriptive sciences, but have been influenced by subjective inputs from the interviewer, which complicates their statistical analysis.

There exists a large literature on avoiding biases in subjective measurements (Arkes 1991, Fischhoff 1982, Hodgkinson et al. 1999, Payne et al. 1999), and many qualitative suggestions have been given. The novelty of our approach is that we propose a quantitative manner for correcting biases in decision under risk and uncertainty when these cannot be avoided. The desirability of such quantitative assessments was pointed out by Fischhoff (1982, pp. 426–427), Viscusi (1995, closing paragraph), and Weber (1994, closing paragraph). Schwartz's (1998) proposal for quantitative corrections of biases in rating scales is similar in spirit to the proposal of this paper.

The central and obviously debatable question is: Which biases and deviations from expected utility do we assume and how do we correct for them (Fischhoff 1982, pp. 423–424)? The analysis of this paper is based on prospect theory and, therefore, we assume two deviations from expected utility: (a) *probability transformation*, the nonlinear weighting of probabilities, and (b) *loss aversion*, the tendency of people to overweight outcomes that are perceived as losses relative to outcomes that are perceived as gains.

Loss aversion designates, in this paper, a deviation from expected utility, depending on psychological perceptions of reference points sensitive to strategically irrelevant reframings of decisions. It is this loss aversion that generates discrepancies between probability- and certainty-equivalent measurements. If there are intrinsic reasons why losses with respect to a status quo are more serious than corresponding gains, then we consider this effect as part of the genuine von Neumann-Morgenstern utility function. It belongs to the expected utility model and does not depend on irrelevant reframings. Our correction proposal concerns only the former loss aversion, which can be expected to occur in quick and dirty data.

A corrective use of prospect theory was suggested before by Fischhoff (1991, p. 839), von Winterfeldt and Edwards (1986, §10.5), and Kahneman and Tversky (1979, p. 277), who wrote:

> These departures from expected utility must lead to normatively unacceptable consequences. ... Such anomalies of preference are normally corrected by the decision maker when he realizes that his preferences are inconsistent. ... In many situations, however, the decision maker does not have the opportunity to discover that his preferences could violate decision rules that he wishes to obey. In these circumstances the anomalies implied by prospect theory are expected to occur.

Several authors have defended the normative status of probability transformation (Machina 1994, Quiggin 1992, Segal 1990, p. 364) and loss aversion (Fischer et al. 1986, pp. 1082–1083). Such views, however, contradict the normative status of expected utility and are, therefore, outside the scope of this paper.

Other deviations from expected utility, such as scale compatibility and the prominence effect (Fischer and Hawkins 1993, Tversky et al. 1988), have been documented. Scale compatibility and the prominence effect predict that certain formats increase the clients' attention for particular aspects of the stimuli, such as high or low outcomes. Given the design of the study that we will present, with both high and low outcomes displayed, such effects are expected to increase noise but not to give rise to systematic biases. A more thorough analysis of these and similar effects is left to future studies.

The importance of reconciling inconsistencies has been widely acknowledged in the literature (Keeney and Raiffa 1976, pp. 198–200, von Winterfeldt and Edwards 1986, §§9.4 and 10.6). Our proposal goes one step further. In our view, inconsistencies are not the essence of the problem; instead they are symptoms. The essence of the problem lies in the biases, i.e., the discrepancies between elicited preferences and the true preferences according to a rational model in which these preferences are to be implemented. Observed inconsistencies prove that biases are present so that corrective procedures are called for. In many utility measurements, however, no consistency checks are made, usually because of time and budget constraints. Biases can be expected to be present as much when consistency checks are carried out to detect them, as when no consistency checks are carried out so that biases remain undetected if present (Birnbaum et al. 1992, p. 333, Kahneman and Tversky 1979, p. 277). We, therefore, propose our corrective formulas also if the data are too limited to uncover inconsistencies, which is usually the case for quick and dirty measurements.

We emphasize that biases and inconsistencies are not to be interpreted as irrationalities on the client's part. Instead, they designate deficiencies in our measurement instruments that, even if the best currently available, do not tap perfectly into the clients' values (Fischhoff 1982, Table 1, Schkade 1998). We are well aware that many of the assumptions underlying our proposal are controversial, such as the very existence of true underlying preferences. These assumptions are, however, the best that we can think of in the current state of the art for situations where decisions *have to* be taken, as good as possible, on the basis of quick and dirty data.

In what follows, §1 presents prospect theory with a varying reference point, the theory upon which the proposals of this paper are based. In §2, we describe probability equivalence (PE) elicitations and certainty equivalence (CE) elicitations, the most common elicitation techniques for risky utility. We discuss how these two techniques should be modified in the presence of probability transformation and loss aversion. Section 3 does the same for the tradeoff (TO) method, an elicitation technique for risky utility introduced by Wakker and Deneffe (1996). They showed that the TO method is robust against probability transformation. Appendix D shows that the method is also robust against a number of distortions generated by reference-point effects. Section 4 summarizes our proposal for modifying the classical evaluation formulas for the PE method, the CE method, and the TO method. It cautions against imprudent use of the PE method, especially at the individual level. If the modified evaluation formulas are a step in the right direction, then the discrepancies between the adjusted PE, CE, and TO utility elicitations can be expected to be reduced. This is tested and verified in §5. Section 6 concludes.

## 1. Prospect Theory

Let $X$ be a set of *outcomes*, elements of which are denoted by $x_1, x_2, \ldots$. We assume that outcomes are real numbers. In the experiment considered in this paper, outcomes designate life duration. They may also designate amounts of money. Because the common utility elicitation procedures invoke only two-outcome gambles, the formal analysis of this paper is restricted to such gambles. A typical *gamble* is denoted by $(p, x; y)$, yielding outcome $x$ with probability $p$ and outcome $y$ with probability $1-p$. If $x = y$, then the gamble is *riskless. Preferences* over gambles are denoted by $\succcurlyeq$. *Strict preferences* are denoted by $\succ$ and *indifferences* by $\sim$. Preferences over outcomes correspond with preferences over riskless gambles. Higher outcomes are preferred to lower outcomes. This implies that utility is strictly increasing in all theories described throughout the paper. As a notational convention, we assume that gambles $(p, x; y)$ are *rank-ordered*, i.e., $x \geq y$. *Expected utility* holds if there exists a function U from the outcomes to the reals, called the *utility function*, such that gambles $(p, x; y)$ are evaluated by $p\mathrm{U}(x) + (1-p)\mathrm{U}(y)$ and preferences and choices correspond with this evaluation.

For descriptive purposes, we consider some deviations from expected utility, described by prospect theory. On our domain of two-outcome gambles, original prospect theory (Kahneman and Tversky 1979) and cumulative prospect theory (Tversky and Kahneman 1992) coincide, and therefore our analysis is valid for both theories. A first deviation from expected utility is that preferences are *sign-dependent*, i.e., they depend on a perceived *reference-point* outcome. The reference point may be the current position ("status quo") of the client. Dependence of preference on the reference point $r$ is expressed by a subscript $r$, as in $\succcurlyeq_r$, $\succ_r$, etc. If $x > r$, then $x$ is a *gain*, and if $x < r$, then $x$ is a *loss*. When no confusion is likely to arise, dependence of preference on $r$ is not expressed in the notation. For now, we assume a fixed reference point $r$. Later, we will consider variations in $r$.

In addition to the utility function for outcomes, prospect theory invokes *probability transformations* $w^+$ and $w^-$ for gains and losses, respectively, and a *loss aversion parameter* $\lambda$. The probability transformations assign 0 to 0 and 1 to 1 and are strictly increasing. The loss aversion parameter is positive. In the formal analysis of Kahneman and Tversky (1979) and Tversky and Kahneman (1992), one fixed reference point is assumed. Then, as a notational convention, the utility of the reference point is assumed zero. Later in this paper, however, we will consider varying reference points, and therefore this notational convention is not followed.

To define the prospect theory functional $\mathrm{PT}(p, x; y)$ upon which the analysis of this paper is based, we first discuss the formalization of loss aversion. Loss aversion has sometimes been incorporated in the utility function (Kahneman and Tversky 1979, Tversky and Kahneman 1992). We separate loss aversion and utility because we will consider varying reference points and because we want to establish a link with expected utility. Therefore, our utility function U describes an intrinsic utility of final wealth. These points also explain why we use the general term *utility function* instead of value function, the more common term in the prospect theory literature. Our method for modeling varying reference points is similar to Shalev's (2000). Like Shalev (p. 272), we do not assume the particular shape of the value function suggested by prospect theory, the purpose of our study being to measure general utility functions (see also Currim and Sarin 1989, p. 24).

If the gamble is *mixed*, i.e., $x > r > y$, then

$$\mathrm{PT}[p, x; y] = \mathrm{U}(r) + w^{+}(p)(\mathrm{U}(x) - \mathrm{U}(r)) - \lambda w^{-}(1-p)(\mathrm{U}(r) - \mathrm{U}(y)). \quad (1)$$

Outcomes are evaluated as deviations from the reference point through terms $\mathrm{U}(x) - \mathrm{U}(r)$ in our model, so as to combine the psychology of prospect theory with the utility function U of expected utility. Numerical perceptions of outcomes (as opposed to intrinsic value), more naturally modeled through terms $v(x-r)$, underly the often-found convex/concave shape of prospect theory's value function as it passes through zero, and are biases from our perspective. Partly, their effects can be modeled through the weighting functions, for another part their modeling is left to future analyses.

If $x \geq y \geq r$, then

$$\begin{aligned}\mathrm{PT}[p, x; y] &= \mathrm{U}(r) + w^{+}(p)(\mathrm{U}(x) - \mathrm{U}(r)) \\ &\quad + (1 - w^{+}(p))(\mathrm{U}(y) - \mathrm{U}(r)) \\ &= w^{+}(p)\mathrm{U}(x) + (1 - w^{+}(p))\mathrm{U}(y). \quad (2)\end{aligned}$$

We finally consider the case $r \geq x \geq y$. The formula is best interpreted as a dual of Equation 2 with $1 - w^{-}(1-p)$ instead of $w^{+}(p)$ and a loss aversion factor ($\lambda$) added to all utility differences with respect to $r$. Unfortunately, the expression of the dual terms requires a complex notation.

$$\begin{aligned}\mathrm{PT}[p, x; y] &= \mathrm{U}(r) - \lambda(1 - w^{-}(1-p))(\mathrm{U}(r) - \mathrm{U}(x)) \\ &\quad - \lambda w^{-}(1-p)(\mathrm{U}(r) - \mathrm{U}(y)) \\ &= \mathrm{U}(r) - \lambda\big(\mathrm{U}(r) - ((1 - w^{-}(1-p))\mathrm{U}(x) \\ &\quad + w^{-}(1-p)\mathrm{U}(y))\big). \quad (3)\end{aligned}$$

Prospect theory reduces to expected utility if $w^{+}(p) = w^{-}(p) = p$ and $\lambda = 1$. Note that the rank-dependent forms in Equations 2 and 3 were also assumed in original prospect theory (Kahneman and Tversky 1979, Equation 2), so that our analysis covers both original and cumulative prospect theory.

We next discuss variation of the reference point. Assume a reference point $r'$ different from $r$. This variation affects which outcomes are perceived as gains and which as losses. Remember that our outcomes refer to final wealth, not to changes with respect to the reference point. There have been no empirical investigations on how the probability transformations $w^{+}$ and $w^{-}$, the utility function U, and the loss aversion parameter $\lambda$ vary with varying reference points. As a working hypothesis, we assume that they do not change systematically, in keeping with the normative requirement of expected utility that U should not depend on reframings affecting the perceived reference point. Schmidt (1999) gives preference axioms for varying reference points. Under the working hypothesis, the above equations can readily be applied with $r'$ instead of $r$.

Empirical studies have shown that the most common pattern of the probability transformation is an inverse S-shape (Abdellaoui 2000, Bleichrodt and Pinto 2000, Gonzalez and Wu 1999, Lattimore et al. 1992, Tversky and Fox 1995, Tversky and Kahneman 1992). This shape implies an overweighting of small probabilities and an underweighting of intermediate and high probabilities. Tversky and Kahneman (1992) proposed the following one-parameter functional form for the probability transformation:

$$w(p) = \frac{p^{\gamma}}{[p^{\gamma} + (1-p)^{\gamma}]^{1/\gamma}}, \quad (4)$$

which has an inverse S-shape for $\gamma$ between 0.27 and 1. They found a median value of $\gamma^{+}$ ($\gamma$ for gains) equal to 0.61 and $\gamma^{-}$ ($\gamma$ for losses) equal to 0.69, and a median value of $\lambda$ equal to 2.25. Further research into probability transformation and loss aversion is still going on today.

## 2. Probability Equivalence and Certainty Equivalence Gambles under Prospect Theory

We assume a maximal outcome $M$ and a minimal outcome $m$. In the experiment described in §5, $m$ corresponds to zero years, i.e., immediate death, and $M$ to 40 years. For monetary experiments, $m$ can be zero if no negative amounts are involved. We normalize utility throughout, i.e., $\mathrm{U}(M) = 1$ and $\mathrm{U}(m) = 0$.

### 2.1. Discrepancy Between Probability Equivalence and Certainty Equivalence

In a PE gamble, a client is faced with two options: an outcome $x$ with certainty and a gamble $(p, M; m)$ where $M \geq x \geq m$. The probability $p$ is varied and the client is asked for which $p$ he is indifferent between the two options. Under expected utility, the resulting *PE indifference*

$$x \sim (p, M; m) \tag{5}$$

gives

$$\mathrm{U}(x) = p\mathrm{U}(M) + (1-p)\mathrm{U}(m) = p. \tag{6}$$

In a CE-gamble, the client is also asked to compare a certain outcome $y$ with a gamble $(q, M; m)$, but now indifference is achieved by varying the outcome $y$. Under expected utility, the resulting *CE indifference*

$$y \sim (q, M; m) \tag{7}$$

gives

$$\mathrm{U}(y) = q\mathrm{U}(M) + (1-q)\mathrm{U}(m) = q. \tag{8}$$

Assume that $q$ in Equation 8 is equal to $p$ in Equation 6. Under expected utility, $y$ in Equation 8 should then be equal to $x$ in Equation 6. Hershey and Schoemaker (1985), however, and many other studies, observed that $y > x$ for most clients (Delquié 1993, Morrison 2000, Johnson and Schkade 1989, Slovic et al. 1990, Wakker and Deneffe 1996). Clients exhibit more risk aversion in PE questions than in CE questions. Expected utility cannot describe the preferences of these clients and other descriptive theories must be invoked. Hershey and Schoemaker hypothesized that this pattern can be explained by a reframing of the PE gamble. We formally analyze their hypothesis in terms of prospect theory. A delicate point in prospect theory is the location of the reference point. This point will be the major concern in the following subsections.

### 2.2. The Probability Equivalence Method

In PE elicitations, the certain outcome $x$ is fixed and, therefore, may provide a salient reference point. This was suggested by Hershey and Schoemaker (1985), Morrison (2000), and Robinson et al. (2001), who interviewed participants about their decision strategies in PE choices. We, therefore, assume the outcome $x$ as reference point in our formal analysis. The PE elicitation thus entails indifference between the reference point $x$ for certain and a mixed gamble yielding a gain $M$ with probability $p$ and a loss $m$ with probability $1-p$. This PE indifference is no longer analyzed by Equation 6 but instead by the prospect theory Equation 1. After some algebraic manipulation, presented in Appendix A, we derive from the PE indifference $x \sim (p, M; m)$, with $\mathrm{U}(M) = 1$ and $\mathrm{U}(m) = 0$:

$$\mathrm{U}(x) = \frac{w^+(p)}{w^+(p) + \lambda w^-(1-p)}. \tag{9}$$

### 2.3. The Certainty Equivalence Method

In the main text and the experiment we analyze the CE method under the assumption that outcomes are perceived as gains. Appendices B and C argue that this is the most plausible case. There we also consider the other cases. As demonstrated in Appendix C, for gains we have

$$\mathrm{U}(y) = w^+(q). \tag{10}$$

To compare Equations 9 and 10, we consider the former in more detail. Empirical research has shown that the loss aversion parameter $\lambda$ exceeds 1 considerably and that $w^-(1-p)$ does not deviate much from $1 - w^+(p)$ for most clients (Bateman et al. 1997, Tversky and Kahneman 1991, 1992). Then the denominator in Equation 9 exceeds 1. For an outcome $x = y$, the probability $p$ in Equation 9 must therefore exceed the probability $q$ in Equation 10, in agreement with the finding of Hershey and Schoemaker (1985) and others.

## 3. The Tradeoff Method

The tradeoff method establishes a sequence of outcomes that are equally spaced in utility units. It was introduced by Wakker and Deneffe (1996) and was analyzed there assuming that all outcomes are gains. We adopt this same assumption in the main text and the analysis of the experiment. Arguments are given in Appendices D and E, where the other cases (i.e., mixed outcomes and pure losses) are also analyzed. These appendices demonstrate that the TO method gives correct utilities not only if probabilities are transformed (as shown by Wakker and Deneffe 1996), but also under the most plausible forms of sign dependence.

The first step in the TO method is to select a *starting outcome* $x_0$ and two *gauge outcomes* $G$ and $g$ such that $G > g$. The value $x_1$ is elicited such that the client is indifferent between the gambles $(p, g; x_1)$ and $(p, G; x_0)$. In agreement with the notational convention of rank-ordered gambles, we assume that the gauge outcomes exceed the other outcomes and, hence, $G > x_0$ and $g > x_1$, thus $G > g > x_1 > x_0$. The probability $p$ and the gauge outcomes $G$ and $g$ are held constant throughout the elicitation process.

After $x_1$ has been elicited, the analyst elicits the outcome $x_2$ such that the client is indifferent between the gambles $(p, g; x_2)$ and $(p, G; x_1)$. This process can be continued as long as the elicited outcomes do not exceed the gauge outcomes, i.e., the rank-ordering of gambles is not affected. A sequence $x_0, \ldots, x_k$ results, where for each $j$ we have the *TO indifference*

$$(p, g; x_j) \sim (p, G; x_{j-1}). \tag{11}$$

We normalize utility such that $U(x_0) = 0$ and $U(x_k) = 1$. As demonstrated in Appendix E, we then get

$$U(x_j) = j/k, \text{ for all } j. \tag{12}$$

## 4. A Proposal for Correcting Biases

We propose to calculate utilities by means of the formulas derived in the preceding sections, when analyzing data from utility elicitation. For instance, assume that a client has stated indifference between \$35K (K = 1000) and (0.70, \$100K; \$0) in a hypothetical PE question. Let utility be normalized such that U(\$100K) = 1 and U(\$0) = 0. Then we propose that

$$U(\$35K) = \frac{w^+(.70)}{w^+(.70) + \lambda w^-(.30)} \tag{13}$$

as in Equation 9 with the parameter values $w^+(0.70)$, $w^-(0.30)$, and $\lambda$ still to be determined. We propose that these utility values are used as inputs in prescriptive decision analyses based on expected utility. Correction for probability transformation was proposed before by Bayoumi and Redelmeier (2000), Birnbaum et al. (1992, e.g., Figure 2), Bleichrodt et al. (1999), Fellner (1961, p. 675), Wakker and Stiggelbout (1995), and Weber (1994).

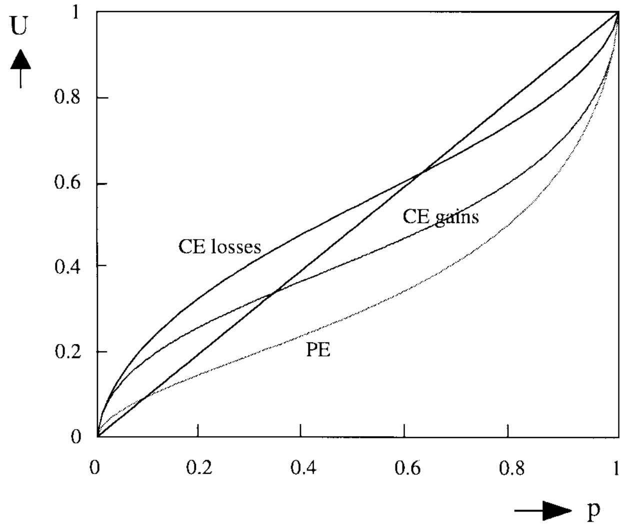
We next discuss the choice of parameter values $w^+(.70)$, $w^-(.30)$, and $\lambda$ in the above example. If the individual parameter values of the client for the relevant outcomes are available, then these values should obviously be used. In the absence of such information and of further empirical evidence, we propose using the estimates found by Tversky and Kahneman (1992), described in §1. There are large individual variations, and the values found by Tversky and Kahneman may substantially deviate from the individual values of the client. Still, at the current state of the art, we think that these median values found by Tversky and Kahneman are better approximations than the values adopted under the classical elicitation assumption (i.e., $w^+(0.70) = 0.70$, $w^-(0.30) = 0.30$, and $\lambda = 1$). Under our proposal, U(\$35K) = 0.42 results.

A number of studies have measured individual utilities of clients under prospect theory for descriptive purposes (Gonzalez and Wu 1999, and the references therein, Tversky and Kahneman 1992). In these studies, many risky choices are observed and then the probability transformation, utility values, and loss aversion parameter if relevant, are chosen to minimize distance from observed choices. Such procedures lead to better descriptive elicitations but are more laborious and take more time. We feel that, if such time is available, prescriptive applications may better resort to interactions with clients than to elaborate observations of choices and parametric fitting.

Figure 1 and Table 1 illustrate the implications for PE elicitations of using Equation 9 with the Tversky and Kahneman estimates. Compared to the classical elicitation assumption, there is a correction upwards from $p = 0.00$ until $p = 0.09$ and a correction downwards from $p = 0.10$ until $p = 1.00$.

The strong corrections near $p = 1.00$ are alarming. For example, the narrow probability interval [0.97, 1.00] serves to measure all utility levels between 0.82 and 1.00. This shows that PE elicitation is particularly insensitive, i.e., it yields little discriminatory

**Figure 1 The Utility Correction Curves**




power, for utilities exceeding 0.80. The above insensitivity is troublesome for decision contexts involving outcomes that are only slightly worse than the maximal outcome $M$, which is a common case in medical decision making. Health states are usually closer to perfect health than to death and, indeed, the PE method usually exhibits low test-retest reliability in medical decision making (Rutten-van Mölken et al. 1995, Stiggelbout and de Haes 2001, p. 224). In addition, the method is usually more difficult for clients than other methods (Dolan et al. 1996, Morrison 2000, Officer and Halter 1968, p. 270, Torrance 1987).

It is not uncommon that half of the clients state that for no probability $p < 1.00$ they are indifferent between a gamble $(p, M; m)$ and outcome $y$ (Lenert et al. 2001, Lundberg et al. 1999). Assuming that clients' maximal accuracy is up to 0.01, such behavior is plausible for any utility of $y$ exceeding 0.95. According to Table 1, $p$ can be expected to exceed 0.995 for such utilities. The difficulties near $p = 1.00$, and to a lesser degree also near $p = 0.00$, are exacerbated by observed irregularities of probability transformations in these regions (Kahneman and Tversky 1979, Tversky and Kahneman 1992, p. 303), inducing additional noise and unreliability for utility estimations.

The insensitivity of PE elicitations is particularly troublesome for utility elicitation at the individual level. Combined with our claim that Table 1 presents the best analysis of PE data given the current state of the art, a negative conclusion must be drawn: We caution against imprudent use of the PE method, especially for applications at the individual level with high-utility outcomes. Therefore, our proposal is primarily intended for policy decisions (Stiggelbout and de Haes 2001, p. 224). Novick and Lindley (1978, p. 308) also cautioned against the PE method with extreme probabilities.

Table 2 provides corrected utilities for the CE method. The table considers the probability levels used in the experiment of this paper. When high probabilities are used, the CE method is insensitive like the PE method. We, therefore, also caution against the use of the CE method with high probabilities. In the majority of certainty equivalent elicitations, probability $q = 0.50$ is adopted and the insensitivity problem does not occur. Equation 10 then yields $U(y) = 0.42$,

**Table 1 Corrected PE Utilities (By Equation 9) as Function of $p$ for $p = 0.00, \ldots, 0.99$; e.g., the Corrected PE Utility for $p = 0.15$ Is 0.123**

| | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.000 | 0.025 | 0.038 | 0.048 | 0.057 | 0.064 | 0.072 | 0.078 | 0.085 | 0.091 |
| 0.1 | 0.097 | 0.102 | 0.108 | 0.113 | 0.118 | 0.123 | 0.128 | 0.133 | 0.138 | 0.143 |
| 0.2 | 0.148 | 0.152 | 0.157 | 0.162 | 0.166 | 0.171 | 0.176 | 0.180 | 0.185 | 0.189 |
| 0.3 | 0.194 | 0.199 | 0.203 | 0.208 | 0.213 | 0.217 | 0.222 | 0.227 | 0.231 | 0.236 |
| 0.4 | 0.241 | 0.246 | 0.251 | 0.256 | 0.261 | 0.266 | 0.271 | 0.276 | 0.281 | 0.286 |
| 0.5 | 0.292 | 0.297 | 0.303 | 0.308 | 0.314 | 0.320 | 0.325 | 0.331 | 0.337 | 0.343 |
| 0.6 | 0.350 | 0.356 | 0.363 | 0.369 | 0.376 | 0.383 | 0.390 | 0.397 | 0.405 | 0.412 |
| 0.7 | 0.420 | 0.428 | 0.436 | 0.445 | 0.454 | 0.463 | 0.472 | 0.481 | 0.491 | 0.502 |
| 0.8 | 0.512 | 0.523 | 0.535 | 0.547 | 0.560 | 0.573 | 0.587 | 0.601 | 0.617 | 0.633 |
| 0.9 | 0.650 | 0.669 | 0.689 | 0.710 | 0.734 | 0.760 | 0.789 | 0.822 | 0.861 | 0.911 |

**Table 2 Corrected CE Utilities**

| Probability $q$ | 0.10 | 0.25 | 0.50 | 0.75 | 0.90 |
|---|---|---|---|---|---|
| CE utility | 0.186 | 0.291 | 0.421 | 0.568 | 0.712 |

generating a correction downwards compared with the classical elicitation assumption. A stated CE value of \$35K for a 50-50 gamble between \$100K and \$0 suggests that U(\$35K) = 0.42.

Because the mentioned insensitivities are particularly troublesome at the individual level, the analysis of the experiment hereafter mainly considers group averages, as relevant for policy making. The application of our proposal to other existing data sets in the literature is an obvious topic for future research.

## 5. Experiment

Our experiment considers hypothetical choices between medical treatments. In experiments, real choices are usually more satisfactory than hypothetical choices. In the health domain with life years and impaired health states as outcomes, however, real-choice experiments can rarely be implemented. Experiments for monetary outcomes suggest that hypothetical and real outcomes give similar results for simple choice situations as considered in this paper (Camerer and Hogarth 1999).

### Participants

The participants were 51 undergraduate economics students from the University of Pompeu Fabra, Barcelona. Participants were paid 5,000 Ptas (approximately \$30 US) to participate in two experimental sessions separated by two weeks. The experiment was administered by personal interview sessions. In the first session, participants answered the TO questions and either the PE or the CE questions. In the second session, participants answered the remaining gamble questions and repeated some questions from the first session to examine the test-retest reliability of their answers. Prior to the main experiment, the questions were tested in pilot sessions using university staff as participants.

**Table 3 The Format of the Tradeoff Questions**

| | Disease 1 | Disease 2 |
|---|---|---|
| Treatment 1 | 55 | $x_{j-1}$ |
| Treatment 2 | 45 | $x_j$ |

### Stimuli of the TO Method

Table 3 illustrates the format of the TO questions. Participants were asked to imagine that they displayed symptoms of one of two diseases. Each disease occurred with probability 0.50. There were two treatments, neutrally described as treatment 1 and treatment 2. The gauge outcomes $G$ and $g$ were set equal to 55 years and 45 years, respectively. The initial outcome $x_0$ was zero years, designating immediate death.

Participants were first given a practice question to become familiar with the TO method. This allowed for a test of whether they had understood the questions. When a participant understood the questions, the actual experiment began. A standard sequence $x_1, \ldots, x_6$ was determined by asking for the value of $x_j$ that established indifference between $(1/2, 45; x_j)$ and $(1/2, 55; x_{j-1})$, $j = 1, \ldots, 6$. Participants were first asked for which values of $x_j$ they had a clear preference. From the remaining values they chose the one for which the alternatives were "most finely balanced."

The second session replicated three TO questions to test the consistency of participants' responses. The three questions that were repeated varied across participants. A practice question preceded the three questions. The procedures in the second session were similar to those in the first.

### Stimuli of the CE Method

In the CE questions, participants were asked to compare a riskless treatment $(1, y)$ with a risky treatment $(p, 40; 0)$. Five CE questions were asked. The probabilities 0.10, 0.25, 0.50, 0.75, and 0.90 were used. This set of probabilities includes both probabilities that are overweighted (0.10 and 0.25) and probabilities that are underweighted according to Tversky and Kahneman's inverse S-shaped probability transformation. The answers to the five CE questions are denoted CE(0.10), CE(0.25), CE(0.50), CE(0.75), and CE(0.90). The order of the CE questions was varied to avoid order effects. Prior to the five CE questions, one practice question was given.

Two CE questions were repeated in the second session for those participants who had answered the CE questions in the first session. The questions that were repeated varied across participants. One practice question preceded the replication questions.

**Stimuli of the PE Method**

Five PE questions were asked in which the riskless treatment yielded 5, 10, 15, 25, and 35 years, respectively. The procedure of administering the PE questions was similar to the procedure used for the CE questions. The outcomes "successful treatment" and "treatment failure" in the risky treatment were set equal to 40 years and 0 years, respectively, the order of the PE questions was varied, and one practice question preceded the PE questions. Two PE questions were repeated in the second session for those participants who had answered the PE questions in the first session. The questions that were repeated varied across participants and were preceded by one practice question.

**Results, Preliminaries**

The data are first analyzed under the classical elicitation assumption, i.e., assuming expected utility. We next consider corrections for probability transformation and loss aversion. Throughout, $U_{TO}$, $U_{CE}$, and $U_{PE}$ denote the utility functions resulting from the TO, the CE, and the PE method, respectively.

Two participants were excluded from the analyses because they refused to make any tradeoffs. Another participant was excluded because his value $x_6$ exceeded the gauge outcome of 45 years so that the analysis of §2 does not apply. Ultimately, 48 participants were included in the analyses. The test-retest reliability of the methods was satisfactory, with nonsignificant differences and high correlations for repeated measurements (PE: 0.88, CE: 0.84, TO: 0.97) for all methods. The results are summarized in Figure 2, and explained in detail in the text that follows. In Figure 2, the favorable case of small discrepancies corresponds with shapes close to the abscissa.

**Comparisons of Different Utility Functions on Common Subdomains**

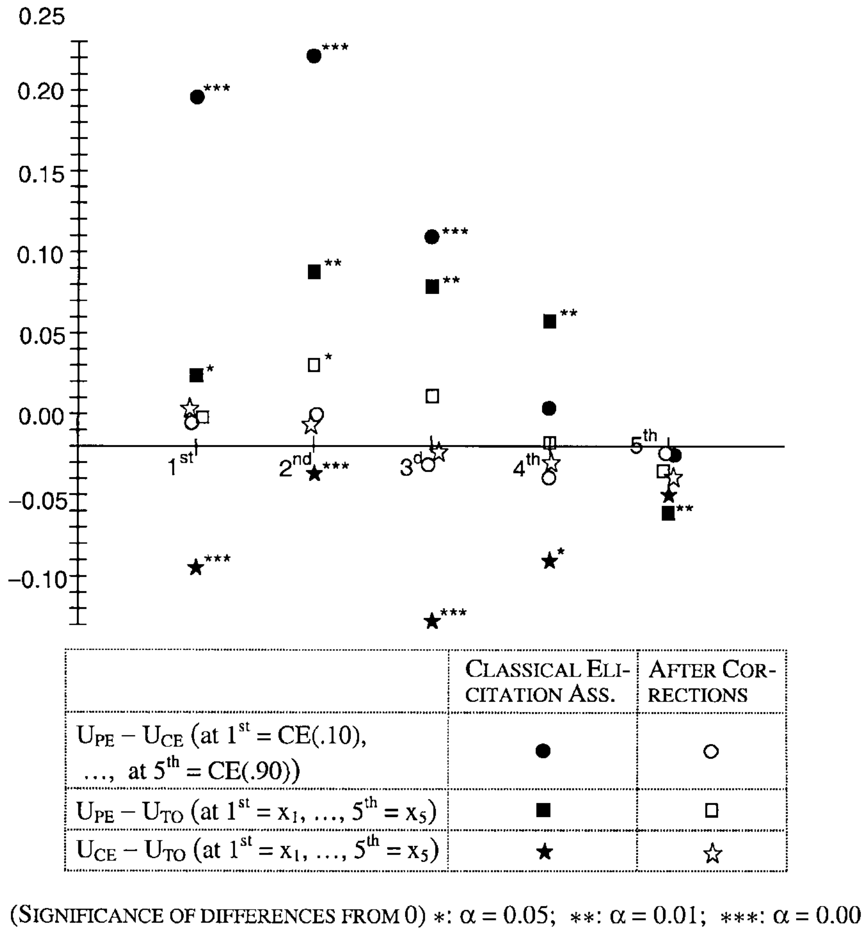
When two utility functions are compared on a common domain, they are usually set zero at the minimal outcome and one at the maximal outcome. Then their curvature (reflected by the Pratt-Arrow measure $-U''/U'$) can be compared. The function with the higher intermediate values is the more concave ("curved downwards").

The highest outcome considered in the TO elicitation, $x_6$, is in general different from 40 years, the highest outcome in the CE and PE elicitations. Therefore, the domains of the utility functions need not coincide, and curvature of utility can be compared only on common subdomains. In such comparisons, utilities are rescaled so as to assign zero to the minimal outcome of the common subdomain and one to the maximal outcome of the common subdomain.

For all but one participant, $x_6$ is smaller than 40, and therefore the common subdomain of TO and PE, and also of TO and CE, is $[0, x_6]$. For the remaining participant, $x_6$ exceeds 40, and the common subdomain is $[0, 40]$. In the former case, $U(0) = 0$ and $U(x_6) = 1$, and we compared the utility functions at $x_1, \ldots, x_5$. For the remaining participant, $U(0) = 0$ and $U(40) = 1$, and we could still compare the utilities at $x_1, \ldots, x_5$ because $x_5$ is less than 40 for this participant. When $U_{CE}$ and $U_{PE}$ are compared with $U_{TO}$, the former two are determined at $x_1, \ldots, x_5$ through linear interpolation. $U_{CE}$ is compared with $U_{PE}$ at CE(0.10), …, CE(0.90), where $U_{PE}$ is determined through linear interpolation. For concave functions, linear interpolation leads to underestimation. Hence, our claims of higher $U_{PE}$ than $U_{TO}$ and higher $U_{PE}$ than $U_{CE}$, made later, are conservative.

The main text reports two-tailed paired $t$ tests. In each case, Wilcoxon signed-ranks tests gave similar results. These tests all concern hypotheses about group averages. Tests of individual differences, which are also mostly reduced after correction, are described in Appendix F. Unfortunately, the latter results are not easy to interpret because of scaling differences of utility before and after correction.

**Figure 2 The Discrepancies Between Utilities With and Without Corrections**




## Results Under the Classical Elicitation Assumption

Table 4 shows the comparisons between the utilities when analyzed under the classical elicitation assumption. In other words, we adopt the prospect theory Equations 1–3 with $w^+$ and $w^-$ equal to the identity function and $\lambda$ equal to one.

There are systematic and significant differences between $U_{PE}$, $U_{CE}$, and $U_{TO}$. $U_{PE}$ exceeds $U_{CE}$ and $U_{TO}$, and $U_{TO}$ exceeds $U_{CE}$. These discrepancies indicate that expected utility is violated and that the classical elicitation assumption does not hold. These results are also illustrated in Figure 2 by the remoteness of the dark shapes from the abscissa.

**Table 4 Comparison Between the Three Methods Under the Classical Elicitation Assumption**

| Question | $U_{PE} - U_{CE}$ ($t_{47}$) | $U_{PE} - U_{TO}$ ($t_{47}$) | $U_{CE} - U_{TO}$ ($t_{47}$) |
|---|---|---|---|
| 1 | 0.215*** (6.97) | 0.043* (2.37) | −0.075*** (−9.47) |
| 2 | 0.241*** (7.41) | 0.108** (3.31) | −0.107*** (−5.36) |
| 3 | 0.130*** (4.13) | 0.099** (3.17) | −0.108*** (−3.83) |
| 4 | 0.023 (0.94) | 0.077** (3.23) | −0.070* (−2.61) |
| 5 | −0.005 (−0.31) | 0.041** (2.70) | −0.030 (−1.70) |
| Total | 0.121 | 0.074 | −0.078 |

$^*p \le 0.05$: $^{**}p \le= 0.01$. $^{***}p \le 0.001$.

## Results after Correction for Probability Transformation and Loss Aversion

We now turn to the analysis of our data by means of prospect theory, i.e., we incorporate not only probabil-

**Table 5 Comparisons Between the Three Methods Under Probability Transformation** ($\gamma^+ = 0.61$, $\gamma^- = 0.69$) **and Loss Aversion** ($\lambda = 2.25$)

| Question | $U_{PE} - U_{CE}$ ($t_{47}$) | $U_{PE} - U_{TO}$ ($t_{47}$) | $U_{CE} - U_{TO}$ ($t_{47}$) |
|---|---|---|---|
| 1 | 0.015 (0.79) | 0.018 (1.37) | 0.022 (1.62) |
| 2 | 0.019 (0.93) | 0.054* (2.41) | 0.013 (0.74) |
| 3 | −0.011 (−0.46) | 0.030 (1.27) | −0.004 (−0.24) |
| 4 | −0.020 (−0.80) | 0.001 (0.03) | −0.011 (−0.64) |
| 5 | −0.004 (−0.17) | −0.015 (−0.95) | −0.019 (−0.13) |
| Total | −0.000 | 0.018 | 0.000 |

$^*p \le 0.05$: $^{**}p \le= 0.01$. $^{***}p \le 0.001$.

ity transformation but also sign-dependence and use the formulas derived in §§1, 2, and 3. Table 5 gives the results.

The table was computed using the estimations obtained by Tversky and Kahneman (1992) ($\gamma^+$, the $\gamma$ parameter of Equation 4 for gains, is 0.61; $\gamma^- = 0.69$; $\lambda = 2.25$). The adjustments for probability transformation and loss aversion have the effects postulated in the theoretical part of this paper: The systematic differences between $U_{PE}$, $U_{CE}$, and $U_{TO}$ vanish. Only the second difference between $U_{PE}$ and $U_{TO}$ is still significant. Figure 2 illustrates these results by white shapes close to the abscissa.

**Further Estimations**

We also analyzed the data under *rank-dependent utility* (Quiggin 1981), i.e., prospect theory without loss aversion and with Equation 2 applied to all gambles with $x \ge y$. In comparison to prospect theory, the CE and TO utilities remain unaffected because they are not subject to loss aversion. The PE utilities are affected; they are now given by $U_{PE}(x_j) = w^+(PE(x_j))$ and come out higher than under prospect theory. Indeed, the resulting PE utilities were significantly higher than the CE and TO utilities (the $t$ statistic varies between 2.41 and 6.52 in the PE-CE comparisons and between 2.77 and 7.02 in the PE-TO comparisons). The remaining discrepancy entails an empirical deviation from rank-dependent utility. We conclude that correcting only for probability transformation is an improvement over the classical elicitation assumption, but not enough to remove the discrepancies. It is also worthwhile to correct for loss aversion.

We estimated the values of $\gamma^+$ and $\lambda$ that best fit our data, in the sense of minimizing squared differences between the elicited utilities, and report median values. Due to a lack of degrees of freedom, we could not estimate $\gamma^-$ as well. The discrepancy between $U_{CE}$ and $U_{TO}$ is minimized for $\gamma^+ = 0.62$. Assuming this value for $\gamma^+$, and $\gamma^- = 0.69$ in agreement with Tversky and Kahneman (1992), the discrepancy between $U_{PE}$ and $U_{CE}$ and between $U_{PE}$ and $U_{TO}$ is minimized for $\lambda = 2.17$ and $\lambda = 3.06$, respectively.

We reanalyzed the data of Wakker and Deneffe (1996) by means of our corrective procedures. The results are similar to those in this paper and are not reported here.

The reconciliation between the different measurements of utility suggest but do not prove that our corrective procedures are in the right direction and lead to a closer approximation of true utility, best representing the interests of the clients. There is no gold standard for utility, and such appropriateness claims for utility therefore have to be speculative. Our speculations are based on prospect theory.

## 6. Conclusion

This paper has used descriptive findings based on prospect theory to improve prescriptive applications of expected utility. In particular, we have argued that loss aversion and probability transformation, two well-documented deviations from expected utility, be recognized and corrected for in utility elicitations.

A first way to deal with deviations from a rational model is trying to avoid them, e.g., by carefully constructing the stimuli (Payne et al. 1999) or by using elicitation procedures that are not affected by the deviations. For instance, the tradeoff method is insensitive to loss aversion and probability transformation. The certainty equivalence method is not distorted by loss aversion, but it is distorted by probability transformation. The probability equivalence method is distorted by both deviations. Unfortunately, the tradeoff method and the certainty equivalence method need continuums of outcomes because they use equivalence matchings in the outcome dimension. These methods are, therefore, not applicable in many contexts, e.g., when evaluating qualitative health states.

Second, if deviations cannot be avoided, they may be detected by means of cross-checkings and subsequently be corrected for interactively with the client. This approach is suggested by the constructive preference approach. Unfortunately, it requires sophisticated interviews and often cannot be implemented because of time and budget constraints.

Third, if no interactive resolutions are possible, then we propose using the best estimations of the deviations available and assuming these when calculating utility. We think that in this way utilities result that better represent the interests of the client than utilities calculated by ignoring the deviations. Our method resolves discrepancies between different elicitation procedures and suggests that the parameters found by Tversky and Kahneman (1992) for monetary outcomes are also good estimations for health outcomes. Our analysis reveals a high insensitivity of the PE method for high-utility outcomes, cautioning against imprudent use of the method.

It is easy to raise methodological and even ethical concerns about our proposal. One may question the normative assumption of expected utility, the assumption of true preferences at all, the mechanical nature of our proposal that at the present stage does not incorporate individual variations, the paternalistic nature of deviating from stated preferences, or, our main concern, the particular biases assumed. Important as these concerns are, for progression of the field, not only should concerns be raised, but also should alternatives and improvements for decision making be advanced. We think that a return to the classical elicitation assumption for the pragmatic "quick and dirty" applications considered here is undesirable. Improvements of the formulas proposed in this paper can be developed if further insights are obtained into the deviations from rational models. In particular, such improvements should be targeted toward the relevant clients and contexts as much as possible.

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## Appendix A: Theoretical Analysis of the PE Method

**Elaboration of Equation 9**

Substituting the prospect theory Equation 1 in the PE indifference $x \sim (p, M; m)$, with $x$ as reference point, implies

$$\mathrm{U}(x) = \mathrm{U}(x) + w^{+}(p)(\mathrm{U}(M) - \mathrm{U}(x)) - \lambda w^{-}(1-p)(\mathrm{U}(x) - \mathrm{U}(m)), \quad \text{(A.1)}$$

or

$$w^{+}(p)(\mathrm{U}(M) - \mathrm{U}(x)) = \lambda w^{-}(1-p)(\mathrm{U}(x) - \mathrm{U}(m)), \quad \text{(A.2)}$$

or

$$w^{+}(p) = (w^{+}(p) + \lambda w^{-}(1-p))\mathrm{U}(x). \quad \text{(A.3)}$$

This implies Equation 9.

## Appendix B: The Reference Point in the CE Method

The analysis of CE elicitations is more difficult because it is not clear what the reference point will be. The certain outcome is not given beforehand and is, therefore, unlikely to serve as reference point (Hershey and Schoemaker 1985). We distinguish the following cases for the CE indifference $y \sim (q, M; m)$, with derivations given in Appendix C.

Case 1. All outcomes are perceived as gains, i.e., the reference point varies within $(-\infty, m]$. Then Equation 10 holds.

Case 2. All outcomes are perceived as losses, i.e., the reference point varies within $[M, \infty)$. Then

$$\mathrm{U}(y) = 1 - w^{-}(1-q). \quad \text{(B.1)}$$

Case 3. Some CE outcomes are perceived as gains and others as losses. Then part of the observed risk aversion is generated by loss aversion. The formula for this case is not tractable and will not be used in our analysis; it is, therefore, not given.

If outcomes are monetary and $m$ is 0, then it is plausible that $m$ is the reference point, i.e., then Case 1 results. Case 2 results if the client takes the other gamble outcome, $M$, as the reference point. The problematic Case 3 is not likely to arise because none of the displayed outcomes, if taken as an anchor, generates it (Hershey and Schoemaker 1985).

For life duration, the outcome used in our experiment, there have not yet been many investigations into clients' choices of reference points. Some indirect evidence suggests that people process life durations as gains. Empirical studies, including the one reported in this paper, generally find risk aversion for life duration, i.e., preference for expected value over a gamble (Gold et al. 1996, Miyamoto and Eraker 1989). For monetary outcomes, risk aversion is predominantly found for gains but not for losses (Tversky and Kahneman 1992). These findings suggest that life durations are processed as gains. Hence, we analyzed the data assuming Case 1 and Equation 10. Then utilities converge after correction for probability transformation and loss aversion (see §5). The results when life durations are processed as losses (Case 2 and Equation B.1) also show convergence of utilities after correction for probability transformation and loss aversion, but to a lesser degree. These results are available from the authors upon request.

## Appendix C: Elaboration of Equations 10 and B.1

Case 1 (all outcomes gains). Substituting the prospect theory Equation 2 in the CE-indifference $y \sim (q, M; m)$ implies

$$U(y) = w^+(q)U(M) + (1 - w^+(q))U(m) \tag{C.1}$$

and thus Equation 10.

Case 2 (all outcomes losses). It can be seen that this case is dual to Case 1 with $1 - w^-(1-q)$ instead of $w^+(q)$ and with the factor $\lambda$ dropping out. Let us, however, give an independent derivation. Using Equation 3 to evaluate the riskless gamble $(1, y)$ gives

$$PT(y) = U(r) - \lambda(U(r) - U(y)). \tag{C.2}$$

Evaluation of $(q, M; m)$ gives

$$\begin{aligned} PT(q, M; m) = {} & U(r) - \lambda w^-(1-q)(U(r) - U(m)) \\ & - \lambda(1 - w^-(1-q))(U(r) - U(M)). \end{aligned} \tag{C.3}$$

Equating these two PT values, dropping common terms $(1-\lambda)U(r)$ from the left- and right-hand sides, and substituting $U(M) = 1$ and $U(m) = 0$, yields $\lambda U(y) = \lambda(1 - w^-(1-q))$. Dropping $\lambda$ yields Equation B.1.

As an aside, let us comment on other outcomes for the CE method. The gamble outcomes in CE elicitations are often not the maximal and minimal outcomes $M$ and $m$ but can be general values $Z$ and $z$. The analysis of this case is analogous to the above analysis. If all outcomes are perceived as gains, then $Z$ and $z$ are substituted for $M$ and $m$ in Equation C.1 and Equation 10 is dropped (unless utility is renormalized to be 0 at $z$ and 1 at $Z$). If all outcomes are perceived as losses then similar modifications apply.

## Appendix D: The Reference Point in the TO Method

We generalize the TO method of Wakker and Deneffe (1996) by considering losses and sign-dependence. We will see that the method remains valid in all plausible cases. Equation 12 holds in the following cases, as will be demonstrated in Appendix E:

Case 1. All outcomes are perceived as gains, i.e., the reference point lies within $(-\infty, x_{j-1}]$ for each TO indifference.

Case 2. All outcomes are perceived as losses, i.e., the reference point lies within $[G, \infty)$ for each TO indifference.

Case 3. All $x_j$ values are perceived as losses, and $g$ and $G$ are perceived as gains, i.e., the reference point lies within $[x_j, g]$ for each TO indifference.

Case 4. All $x_j$ values and $g$ are perceived as losses, $G$ is perceived as a gain, the reference point lies within $[g, G]$ and is the same for all TO indifferences.

In Cases 1, 2, and 3 the reference point can vary from one TO indifference to the other as long as it remains within the specified domain. In Case 4, the reference point should remain fixed. The most likely scenario is that the client anchors on one of the prespecified values in the TO question in Equation 12 and takes that as reference point. Case 1 covers the choice $x_{j-1}$, Case 3 the choice $g$, and Case 2 the choice $G$. (Choices $g$ and $G$ are also covered by Case 4.) Case 2 results if life expectancy is taken as the reference point in the experiment described in §5.

The TO method fails when (i) the reference point varies between $g$ and $G$, (ii) the reference point falls between $x_{j-1}$ and $x_j$ in some TO indifference (Equation 11), and (iii) the reference point varies more extremely.

In case (i), the bias in Equation 12 can lead to over- or underestimation of utility and will be nonsystematic. Most other cases lead to overly concave utility because then the risk aversion generated by loss aversion is modeled through the utility function. The cases (i), (ii), and (iii) do not seem likely to arise, and therefore the TO method will usually yield valid utility inferences.

We have analyzed the TO method for the case where $G > g \geq x_k > \cdots > x_0$. A similar analysis applies to the case where $x_k > \cdots > x_0 \geq G > g$. Distortions generated by sign-dependence are also implausible in this case, and Equation 12 will still hold. The cases where $x_0 > \cdots > x_k \geq g > G$ or $g > G \geq x_0 > \cdots > x_k$ are analyzed similarly and lead to utilities $U(x_j) = (k-j)/k$.

## Appendix E: Elaboration of Equation 12

Consider the indifference $(p, g; x_1) \sim (p, G; x_0)$. We first give a generic formula for prospect theory and then explain that it holds for Cases 1–3 in Appendix D. The generic formula for prospect theory is

$$\begin{aligned} & \alpha(U(g) - U(r)) + \beta(U(x_1) - U(r)) \\ & \quad = \alpha(U(G) - U(r)) + \beta(U(x_0) - U(r)). \end{aligned} \tag{E.1}$$

If $G$ and $g$ are both gains (Cases 1 and 3), then $\alpha$ is $w^+(p)$; if $G$ and $g$ are both losses (Case 2), then $\alpha$ is $\lambda(1 - w^-(1-p))$. If $x_1$ and $x_0$ are both gains (Case 1), then $\beta$ is $1 - w^+(p)$; if $x_1$ and $x_0$ are both losses (Cases 2 and 3), then $\beta$ is $\lambda w^-(1-p)$. Equation E.1 follows in each case from substituting Equations 1–3. We leave these substitutions to the reader and only note that for gains $x$ the same weights apply in Equations 1 and 2, and for losses $y$ the same

**Table 6 Discrepancy Before Correction Minus Discrepancy After Correction**

| Question | PE vs. CE ($t_{47}$) | PE vs. TO ($t_{47}$) | CE vs. TO ($t_{47}$) |
|---|---|---|---|
| 1 | 0.120*** (4.92) | 0.028*** (3.79) | 0.006 (0.55) |
| 2 | 0.151*** (6.19) | 0.071*** (5.57) | 0.063*** (5.02) |
| 3 | 0.093*** (5.53) | 0.069*** (4.70) | 0.093*** (6.94) |
| 4 | 0.006 (0.68) | 0.038* (2.53) | 0.074*** (5.44) |
| 5 | −0.051*** (−6.94) | 0.013 (1.21) | 0.033** (3.13) |

*$p \leq 0.05$: **$p \leq= 0.01$. ***$p \leq 0.001$.

weights apply in Equations 1 and 3. Common terms U($r$) can be canceled.

Let us first analyze the TO method for the cases where Equation E.1 applies, which covers Cases 1–3. Algebraic manipulation yields

$$\mathrm{U}(x_1) - \mathrm{U}(x_0) = \frac{\alpha}{\beta}[\mathrm{U}(G) - \mathrm{U}(g)]. \tag{E.2}$$

Similarly, the indifference

$$(p, g; x_j) \sim (p, G; x_{j-1}) \tag{E.3}$$

implies, for each $j$,

$$\mathrm{U}(x_j) - \mathrm{U}(x_{j-1}) = \frac{\alpha}{\beta}[\mathrm{U}(G) - \mathrm{U}(g)]. \tag{E.4}$$

Combining Equations E.2 and E.3 gives

$$\mathrm{U}(x_j) - \mathrm{U}(x_{j-1}) = \mathrm{U}(x_1) - \mathrm{U}(x_0). \tag{E.5}$$

This implies Equation 12. Because the reference point cancels from Equation E.1, variations in the reference point from one TO question to the other are permitted as long as the gain/loss status of the outcomes $G$, $g$, and the $x_j$s is not affected.

For Case 4 in Appendix D (reference point between $g$ and $G$), the preceding derivations are adjusted as follows. The left-hand side $\alpha$ in Equation E.1 is replaced by $\lambda(1 - w^-(1-p))$ and the right-hand side $\alpha$ by $w^+(p)$. $\beta$ is $\lambda w^-(1-p)$. The right-hand sides in Equations E.2 and E.4 are then replaced by

$$\frac{w^+(p)(\mathrm{U}(G) - \mathrm{U}(r)) - \lambda(1 - w^-(1-p))(\mathrm{U}(g) - \mathrm{U}(r))}{\beta}. \tag{E.6}$$

Equations E.5 and 12 still hold. Note, however, that $r$ does not cancel in Equation E.6 and, therefore $r$ must remain fixed in this case.

## Appendix F: Analysis of Individual Data

This appendix reports a test of individual differences. It is well known that preference elicitations exhibit large deviations between individuals and contain much noise. Consequently, some authors avoid analyzing preference data at the individual level (Torrance 1986, p. 27, Tversky and Kahneman 1992, §2.2). In our case, there is a further reason for not presenting the individual analysis in the main text, explained hereafter.

For each individual, we calculated the *discrepancies* (absolute values of differences) between the various utility assessments both before and after correction. Table 6 presents two-tailed paired $t$ tests comparing these discrepancies. (Wilcoxon signed-ranks tests gave similar results.) All discrepancies, except for the fifth test of PE versus CE, were reduced after correction.

Unfortunately, there is no easy interpretation of these findings because comparisons of utility differences before and after correction are distorted by "local" changes in scaling—no such comparisons were used in the group analyses presented in the main text. For values in the middle of the range, our correction formulas compress utilities toward 1/2 and this, rather than a convergence toward a true underlying utility, may explain the reduction of discrepancies. At the upper end of the scale, our correction function is steep, and therefore discrepancies are enlarged. This, rather than a failure of our method, may explain the PE-CE comparison at the fifth value. If a detailed error theory for the participants' behavior were assumed, then we could calculate the discrepancies predicted by expected utility and by prospect theory and compare these in a meaningful manner. There exist, however, only a few error theories for individual decisions (Buschena and Zilberman 2000, Harless and Camerer 1994, Hey and Orme 1994) and it is not clear which one would be appropriate for our data. We, therefore, leave this as a topic for future research.

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