Treatment decisions under ambiguity

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1. Introduction

Two seminal articles in the second half of the 1970s showed how accounting for people’s attitudes towards risk can improve the practice of health economics and medical decision making (McNeil et al., 1978; Pauker and Kassirer, 1975). These studies were based on expected utility, which was the dominant descriptive theory of decision under risk at the time. Later studies showed how their recommendations could be improved using new insights from decision theory, in particular prospect theory (Wakker, 2008, 2010).

Studies of medical decision making typically assume that probabilities are known. In real-life situations, however, these are often unknown and the available information is given with different degrees of precision. There is considerable uncertainty about the risks to public health that we face. Examples are the recent debates about the threats of mad cow disease, climate change, and the avian and swine flu. Similarly, doctors face uncertainty in making treatment recommendations. Data on the prevalence of disease and on the success rate of treatment are incomplete or unknown and the available data do not allow extracting a single probability distribution of the possible outcomes (see Arad and Gayer, 2012 for an example involving medical decisions).

In health economics, ambiguity has usually been addressed by sensitivity analysis (Briggs et al., 1994; Manski, 2011) or by meta-analyses in which the different probabilities found in the literature are combined using a weighted average of the available estimates. These approaches implicitly assume that the decision maker is neutral towards ambiguity. However, an extensive amount of empirical work, originating from Ellsberg’s (1961) famous thought experiment, show that people are not neutral towards ambiguity, but dislike ambiguity and are ambiguity-averse (for examples regarding medical decisions see Curley et al., 1989; Han et al., 2009; Portnoy et al., 2011). Ignoring this ambiguity aversion may distort treatment recommendations and may hinder the understanding of variations in treatment practice.

Many new models have been developed to capture ambiguity aversion (Ghirardato et al., 2004; Gilboa and Schmeidler, 1989; Klibanoff et al., 2005; Maccheroni et al., 2006; Schmeidler, 1989; Tversky and Kahneman, 1992). These models have found only limited application in health economics. The purpose of this paper is to explore the implications of ambiguity aversion for treatment decisions. We study the impact of ambiguity aversion on two classical problems in medical decision making under the smooth ambiguity model of Klibanoff et al. (2005). This model introduces a simple and easily interpretable way to capture ambiguity aversion.
and it is popular in economics today (Gollier, 2011; Treich, 2010). However, our main results also hold under other ambiguity models and we will briefly discuss these in the concluding section.

The main text will present the results in an intuitive, graphical way to explain the main ideas and concepts involved. Extensions and formal proofs of these intuitive results are provided in the appendix. In Section 2, we start by incorporating ambiguity into Pauker and Kassirer's (1975) model of diagnostic choice where the prevalence of the disease is unknown (diagnostic ambiguity). We show that in this model ambiguity aversion leads to an increase in the propensity to treat. It has been argued that ambiguity aversion is irrational and a bias in human decision making (Wakker, 2010). If so, ambiguity aversion leads to a welfare loss. We consider these welfare costs in Section 3. In Sections 4 and 5, we study the case where the effects of treatment are ambiguous (therapeutic ambiguity). In this case, the effects of ambiguity aversion are reversed and ambiguity aversion reduces the propensity to treat. Section 6 concludes the paper and discusses our main findings.

2. Diagnostic ambiguity

Consider a patient who displays particular symptoms. The decision maker, who could be the doctor, the patient, a policy maker or someone else, has to decide whether the patient should undergo treatment. The treatment decision has to be made before the decision maker knows the true health state of the patient. We assume that there are two possible health states: either the patient is sick (s) or he is healthy (h). The decision maker can only decide between treatment (T) and no treatment (NT). The case where the decision maker can also decide on the intensity of treatment is considered in Appendix A.2.

Let $H^s_h$ [$H^T_h$] denote the patient's health when he is treated and he turns out to be healthy [sick]. Likewise, $H^s_{NT}$ [$H^T_{NT}$] is the patient's health when he is not treated and turns out to be healthy [sick]. We assume that health can be quantified, for example as the number of remaining (quality-adjusted) life-years, and that $H^s_{NT} > H^s_h > H^T_h$. In other words, treatment is beneficial when sick, but detrimental when healthy, and it is always better to be healthy than to be sick. The outcomes of the treatment are known. The case of therapeutic hazard is considered in Sections 4 and 5.

The probability that the patient is sick is ambiguous. To explain the intuition underlying our general result, we assume that this probability can take two values, $p_1$ and $p_2$ with $p_1 < p_2$. The case where the probability can take on more than two values is treated in Appendix A.1, which also contains a formal proof of the intuitive results derived in this section. Based on the information at his disposal, the decision maker assigns beliefs to the probability of illness. Let $\mu$ denotes the decision maker's subjective probability that $p_1$ is the true probability of illness. Consequently, $1 - \mu$ is his belief that the true probability of illness is $p_2$.

To study the impact of ambiguous beliefs, we assume that the decision maker behaves according to the smooth ambiguity model of Klibanoff et al. (2005) (KMM). Then, his utility of treatment is equal to

$$V^T = \mu \psi(p_1) U(H^T_h) + (1 - \mu) \psi(p_2) U(H^T_h) + (1 - p_2) U(H^s_h).$$

And his utility of no treatment is

$$V^{NT} = \mu \psi(p_1) U(H^s_{NT}) + (1 - \mu) \psi(p_2) U(H^T_{NT}) + (1 - p_2) U(H^s_{NT}).$$

In Eqs. (1) and (2), $U$ is a von Neumann Morgenstern utility function over health. We assume that the decision maker prefers more health to less, i.e., $U$ is strictly increasing ($U > 0$) and that he is averse to risk, i.e., $U$ is strictly concave ($U'' < 0$). An attractive feature of the smooth ambiguity model is that it separates a decision maker's ambiguity, measured through the probabilities $\mu$ and $1 - \mu$, and his ambiguity aversion, measured through the function $\psi$. The decision maker is ambiguity averse if $\psi$ is concave and ambiguity seeking if $\psi$ is convex. Ambiguity neutrality, the case usually assumed in medical decision making, corresponds with linearity of $\psi$.

The smooth ambiguity model can be interpreted as a two-stage model in which the first stage determines the probability of illness ($p_1$ or $p_2$) and the second stage determines whether the patient is healthy or sick. In each stage, the decision maker uses an expected utility evaluation, but the utility function that is used in the two stages differs. In the first stage, the decision maker uses $\psi$, which reflects his ambiguity aversion, whereas in the second stage, he uses $U$, which reflects his risk aversion.

Let $EU^T_{p_1}$, $EU^T_{p_2}$ denote the expected utility of treatment [no treatment] when the expected probability of illness is equal to $p$. Whatever the decision taken, expected utility is a decreasing function of the expected probability of illness. It decreases linearly because the expected utility model is linear in probability. The behavior of expected utility as a function of $p$ is illustrated in Fig. 1. The $EU^T_{p_1}$ line is less steep than the $EU^T_{p_2}$ line. This feature will be important in the sequel and it results from the fact that the utilities of the potential outcomes under treatment ($H^T_h$ and $H^T_h$) are closer together than the utilities of the potential outcomes under no treatment ($H^s_{NT}$ and $H^T_{NT}$). Consequently, for a given level of ambiguity around the probability of illness ($p_1$ and $p_2$ in Fig. 1), the spread of the expected utilities is less under treatment than under no treatment: $EU^T_{p_1} - EU^T_{p_2}$ is smaller than $EU^{NT}_{p_1} - EU^{NT}_{p_2}$. For
low probabilities of illness, no treatment is better than treatment and for high probabilities of illness, treatment is better than no treatment. $EU^T_p$ and $EU^{NT}_p$ cross at $\hat{p}$. Because an ambiguity-neutral decision maker has linear $\varphi$ and behaves according to expected utility, $\hat{p}$ is the probability of illness at which an ambiguity neutral decision maker is indifferent between treatment and no treatment.

Let us now consider what happens under ambiguity aversion. Fig. 2 illustrates the function $\varphi$ which is defined over the different expected utilities. The straight line depicts the situation under ambiguity neutrality ($\varphi$ linear). Without loss of generality, we have scaled $\varphi$ such that $\varphi(EU^{NT}_{p_2}) = EU^{NT}_{p_2}$ and $\varphi(EU^{NT}_{p_1}) = EU^{NT}_{p_1}$.

Fig. 2 shows that ambiguity aversion (concavity of $\varphi$) implies that $\varphi(EU^T_{p_2}) > EU^T_{p_1}$ and that $\varphi(EU^{NT}_{p_2}) > EU^{NT}_{p_1}$. This means, in turn, that $\varphi(EU^T_{p_2}) - \varphi(EU^{NT}_{p_2})$ exceeds $EU^T_{p_2} - EU^{NT}_{p_2}$ and that $\varphi(EU^{NT}_{p_1}) - \varphi(EU^T_{p_1})$ falls short of $EU^{NT}_{p_1} - EU^T_{p_1}$. Compared with ambiguity neutrality, ambiguity aversion makes the advantage of treatment over no treatment (the perceived difference between $EU^T_{p_2}$ and $EU^{NT}_{p_2}$) more salient relative to the advantage of no treatment over treatment (the perceived difference between $EU^{NT}_{p_1}$ and $EU^T_{p_1}$). Consequently, ambiguity aversion increases the likelihood that treatment is preferred to no treatment.

We can now reproduce Fig. 1 under ambiguity aversion (see Fig. 3). The effect of ambiguity aversion is to shift the line displaying the benefits of treatment upwards, while the line displaying the benefits of no treatment is unaffected (by the chosen scaling of $\varphi$). Fig. 3 shows that an ambiguity averse decision maker prefers treatment at $\hat{p}$, the expected probability of illness at which an ambiguity neutral decision maker is indifferent between treatment and no treatment. Indifference between treatment and no treatment is restored at the lower probability of illness $\hat{p}$. An ambiguity averse decision maker will sooner opt for treatment, hence, ambiguity aversion increases the propensity to treat.

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2 This scaling is allowed by the uniqueness properties of $\varphi$.
in which there is no ambiguity aversion and agents are ambiguity neutral. In our analyses, the cost of treatment played no role and was implicitly assumed to be zero. As illustrated by Fig. 1, agents will choose treatment if the expected probability of illness is sufficiently large. Moreover, because the utility of treatment exceeds the utility of no treatment, agents will still choose treatment for a positive cost. However, if the cost of treatment becomes too high, \( \hat{c} \) in Fig. 4, agents will no longer choose treatment and prefer to go untreated. Hence, the demand for treatment as a function of the cost of treatment is a step function as illustrated by the solid line in Fig. 4.

Suppose next that agents are ambiguity averse. As we saw in Section 2, ambiguity aversion increases the propensity to choose treatment and the maximum cost of treatment for which ambiguity averse agents opt for treatment will rise to \( \tilde{c} \). The demand curve (which is still a step function) will shift upwards as illustrated by the interrupted curve in Fig. 4. Now, if the true cost of treatment exceeds \( \tilde{c} \) or falls short of \( \hat{c} \) ambiguity aversion does not lead to a welfare loss. In the first case, ambiguity aversion is not strong enough to entice agents to opt for treatment. In the second case, ambiguity neutral agents already chose treatment and the introduction of ambiguity aversion only reinforces their preference. However, if the true cost of treatment is between \( \tilde{c} \) and \( \hat{c} \), a welfare loss equal to the shaded area in the figure occurs. Ambiguity averse agents are willing to pay this cost of treatment, but the true (normative) valuation of treatment is given by the ambiguity neutral demand curve and this is less than the cost of treatment.

In real life, agents are not homogeneous and their attitudes towards ambiguity will vary. Fig. 5 illustrates the effect of heterogeneous ambiguity aversion on the demand for treatment. If there is heterogeneity in ambiguity aversion, it will no longer be true that all agents switch from treatment to no treatment at the same cost \( \hat{c} \). Instead, the switch from treatment to no treatment will be a gradual process. First the least ambiguity averse agents will opt for no treatment and, as the cost of treatment rises further, more and more agents will opt for no treatment until the cost of treatment is so high that only extremely ambiguity averse agents still opt for treatment. Hence, the demand curve will be downward sloping up till point \( \tilde{c} \). For simplicity, we have drawn the demand for treatment as a linear function of the cost of treatment although in reality it will probably have a more jagged character. As in Fig. 4, if the true cost of treatment falls short of \( \tilde{c} \), heterogeneous ambiguity aversion leads to no welfare loss, because ambiguity neutral agents would have chosen the same level of treatment. If the true cost of treatment is between \( \tilde{c} \) and \( \hat{c} \), the welfare loss is equal to the shaded area in Panel A of Fig. 5, which is smaller than the welfare loss in the case of homogeneous ambiguity aversion. This is so because some agents (the least ambiguity averse) do not opt for treatment at this cost whereas all homogeneous agents opted for treatment. On the other hand, if the cost of treatment exceeds \( \hat{c} \), there is still a welfare loss in the heterogeneous case (equal to the dotted area in Panel B) because the most ambiguity averse agents will still choose treatment.

4. Therapeutic ambiguity

In the previous two sections, the only source of ambiguity was the probability of illness. The effects of treatment were known with certainty. In this section, we will analyze the case where the effects of treatment are ambiguous.

We consider a model that was introduced by [source]. Assume that there is no diagnostic ambiguity. The decision maker knows for sure that the patient is ill so that in the absence of treatment, the patient’s health is \( H^{NT} \). The effects of treatment are, however, ambiguous. An example is the situation in which a physician is unsure about the mortality risk of a specific kind of surgery. Different medical studies may have reported different mortality rates and the physician is unsure about the correct rate. Or, alternatively, the mortality rate may depend on patient characteristics, which are unobservable for the physician.

Let \( H^{L+} \) and \( H^{L-} \) denote the patient’s health if treatment is successful and not successful, respectively. We will assume throughout that \( H^{L-} < H^{NT} < H^{L+} \). In words, successful treatment is beneficial for the patient, but if treatment fails, he ends up in worse health than if he were left untreated. The decision maker is unsure about the probability that treatment will fail and believes that it can take two values, \( p_1 \) and \( p_2 \) with \( p_1 < p_2 \). The more general case where the set of possible failure rates \( p \) consists of more than two values will
be considered in Appendix A.3 where we also give a formal proof of the results presented intuitively in this section. By $\mu$ we denote the decision maker’s subjective belief that the probability of treatment failure is $p_1$ and thus $1 - \mu$ is his belief that the probability of treatment failure is $p_2$.

According to the smooth ambiguity model, the utility of treatment is equal to:

$$V^T = \mu \varphi(p_1 U(H_{h}^{T+}) + (1 - p_1)U(H_{h}^{T-})) + (1 - \mu) \varphi(p_2 U(H_{h}^{T+}) + (1 - p_2)U(H_{h}^{T-})) \tag{3}$$

And the utility of no treatment is equal to:

$$V^{NT} = \varphi(U(H_{s}^{NT})) \tag{4}$$

Assume that an ambiguity neutral decision maker is indifferent between treatment and no treatment at expected probability $\bar{p}$:

$$\bar{p} U(H_{h}^{T+}) + (1 - \bar{p}) U(H_{h}^{T-}) = U(H_{s}^{NT}) \tag{5}$$

Fig. 6 illustrates the case of an ambiguity-neutral decision maker. Let $EU_{p_{1}}$ $EU_{p_{2}}$ $EU_{NT}$ denote the expected utility of treatment [no treatment] when the failure rate of treatment is $p$. $EU_{NT}$ is equal to $U(H_{s}^{NT})$ and, hence, it is constant and does not depend on $p$. This is illustrated by the horizontal light line in Fig. 6. Because $EU_{NT}$ is constant, we will simply write $EU_{NT}$ from now on. The expected utility of treatment $EU_{p_{1}}$ decreases with the failure rate of treatment and is equal to $U(H_{s}^{NT})$ when treatment fails for sure ($p = 1$) and to $U(H_{h}^{NT})$ when there is no risk of treatment failure ($p = 0$).

Fig. 7 shows the effect of ambiguity aversion ($\varphi$ concave). Ambiguity aversion increases the attractiveness of no treatment relative to treatment. Without loss of generality, we scaled $\varphi$ such that $\varphi(EU_{p_{1}^{T+}}) = EU_{p_{1}^{T+}}$ and $\varphi(EU_{p_{2}^{T+}}) = EU_{p_{2}^{T+}}$. Concavity of $\varphi$ then leads to an increase in the benefits of no treatment, $\varphi(EU_{NT}) > EU_{NT}$, while the benefits of treatment remain constant.

Because ambiguity aversion increases the attractiveness of no treatment, the line depicting the value of no treatment shifts upwards and the decision maker now prefers no treatment to treatment at the failure rate $\bar{p}$ (see Fig. 8). The ambiguity averse decision maker is less willing to accept treatment failure. He will now be indifferent between treatment and no treatment for a lower failure rate ($\bar{p}$).

5. The welfare costs of therapeutic ambiguity aversion

Fig. 9 shows the welfare costs of therapeutic ambiguity aversion when all agents are homogeneous. The solid curve shows the demand for treatment under ambiguity neutrality. Like in Fig. 4, if the failure rate of treatment is sufficiently low, agents will be willing to pay a cost of $\bar{c}$ for treatment. If the cost of treatment exceeds $\bar{c}$, the demand for treatment drops to zero.

As we saw in Section 4, ambiguity aversion makes decision makers less inclined to choose treatment. Hence, the effect of ambiguity aversion is to shift the demand curve for treatment downwards: the maximum cost agents are willing to pay drops from $\bar{c}$ to $\bar{c}$. There
6. Discussion

In many medical decisions reliable information about the risks involved is lacking. Empirical studies have shown that people dislike such ambiguity and often react strongly to it. An example from public health is the overreaction to the risks posed by the swine flu (a case of diagnostic ambiguity) and to the vaccine used against it (a case of therapeutic ambiguity). It is plausible that similar overreactions take place in clinical practice and that differences in ambiguity aversion contribute to observed differences in treatment practice. Many new theories of ambiguity aversion have been proposed and the study of ambiguity is currently a central topic in economics and decision theory. In spite of this, ambiguity has largely been ignored in health economics and medical decision making, which still rely on the tools of evidence-based medicine and sensitivity analysis.3

We have shown in two medical decision problems that an increase in ambiguity entails a cost for an ambiguity averse decision maker in the sense that he deviates from his optimal choice in the absence of ambiguity. This is much like an increase in risk entails a cost for a risk averse decision maker. In the problems we considered, ambiguity aversion reinforces risk aversion and leads to an increase in the propensity to treat in the case of diagnostic risk and to a decrease in the propensity to treat in the case of therapeutic risk. Ambiguity aversion does not always reinforce risk aversion. Gollier (2011) showed that there are situations in which ambiguity aversion and risk aversion go in opposite directions. In our models, this did not occur because we analyzed the case where there are two states of the world (sick versus healthy in the case of diagnostic ambiguity and treatment success versus treatment failure in the case of therapeutic ambiguity). It is of interest to explore the effects of ambiguity aversion when there are more than two states of nature.

The implications of our results depend on whether ambiguity aversion is seen as rational or not. The literature is divided on this. If ambiguity aversion is rational, our analyses show the distortion in treatment recommendations when it is ignored. On the other hand, if ambiguity aversion is considered irrational, our analyses (and in particular Sections 3 and 5) show the welfare loss resulting from ambiguity aversion.

We assumed that ambiguity aversion could be modeled by the smooth ambiguity model. This model, while increasingly popular in economics, has recently been criticized (Bailon et al., 2012b; Epstein, 2010). Therefore, we have also studied the effects of diagnostic ambiguity and therapeutic ambiguity under other ambiguity models.4 Our conclusions about the effects of ambiguity aversion under diagnostic and therapeutic ambiguity hold in all these models.5 To illustrate, the analysis under prospect theory (Tversky and Kahneman, 1992), the main descriptive alternative for the smooth ambiguity model, is presented in Appendix A.4.

Our paper can be extended in several directions. One obvious extension is to simultaneously study the effects of diagnostic ambiguity and therapeutic ambiguity. In real-world decisions these two types of ambiguity often occur jointly. To model this, the utility of treatment for a specific probability of illness as expressed by Eq. (3) has to be substituted into Eq. (1). The resulting expression is complex and difficult to analyze because it involves two types of ambiguity which go in opposite directions. Nevertheless, our analysis provides some guidance on the overall effects of diagnostic ambiguity and therapeutic ambiguity. In general, the outcome of these opposing forces will depend on three factors. The first is the degree of diagnostic and therapeutic ambiguity. Our analysis reveals that the more ambiguity there is (reflected by the difference between p1 and p2 in Figs. 1 and 6) the stronger the impact of ambiguity aversion is. So if, for instance, we assume

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Footnotes:
4 An exception is the recent work by Paul Han and co-authors (Han et al., 2009, 2011).
5 In particular, we studied the impact of diagnostic ambiguity and therapeutic ambiguity under maxmin expected utility (Gilboa and Schmeidler, 1989), α-maxmin expected utility (Eckehoudt and Jelewa, 2004; Ghirardato et al., 2004; Jaffray, 1989), contraction expected utility (Gajdos et al., 2008), and prospect theory (Tversky and Kahneman, 1992).

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The results of these analyses can be found at http://people.few.eur.nl/bleichrodt/Results_under_other_ambiguity_models.pdf.
there is more diagnostic ambiguity than therapeutic ambiguity, ambiguity aversion will tend to increase the decision maker’s propensity to opt for treatment. Likewise, if therapeutic ambiguity dominates diagnostic ambiguity, the decision maker will be inclined not to treat.

The second factor that plays a role is the dispersion of the outcomes of treatment and no treatment. If, for example, in the case of therapeutic ambiguity, the negative effects of treatment failure are small, the slope of the treatment line in Fig. 6 will be relatively flat and ambiguity has little effect on the utility of treatment, and the decision maker will be more likely to opt for treatment.

Finally, the decision maker might have a different attitude to diagnostic ambiguity than to therapeutic ambiguity. If the decision maker is more averse to diagnostic ambiguity than to therapeutic ambiguity, he will be more likely to treat. Conversely, if he is more averse to therapeutic ambiguity, the decision maker is more likely not to treat. Empirical evidence suggests that people have different attitudes to different sources of uncertainty (Abdellaoui et al., 2011; Kilka and Weber, 2001; Tversky and Wakker, 1995).

The smooth model does not allow for such source preference as it uses one function \( \psi \) which applies to all sources of uncertainty. In the literature, there are similar models to the smooth model that do allow for such source preference (Chew et al., 2008; Ergin and Gul, 2009). Likewise, prospect theory can account for source preference (Fox and Tversky, 1998). Whether source-dependence also applies to medical decisions is a topic worthy of future research.

To summarize the above, if there is both diagnostic and therapeutic ambiguity, treatment is more likely if (a) there is more diagnostic ambiguity than therapeutic ambiguity; (b) the spread in the outcomes of treatment is small relative to the spread in the outcomes of no treatment; (c) the decision maker is more averse to diagnostic ambiguity than to therapeutic ambiguity.

Another potentially fruitful area of further exploration is to study the impact of background sources of uncertainty. It is known from the theory of decision under risk that the presence of background risks can substantially affect optimal behavior and has led to the important notion of prudence (Bui et al., 2005; Courbage and Rey, 2006, 2012; Kimball, 1990). It is of interest to explore whether similar effects occur under ambiguity.

Ambiguity is a rich field and its implications have only been partially understood. We hope that our paper will prove useful in showing how its impact on medical decisions can be modeled and that it will pave the way for further studies of ambiguity in health economics.

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Appendix A. Extensions and proofs

A.1. Diagnostic ambiguity with more than two beliefs

In this appendix, we show that the conclusion that ambiguity aversion leads to an increase in the decision maker’s propensity to treat compared with ambiguity neutrality holds for a finite set of possible probabilities of illness. Suppose that there are \( n \) probabilities in the set of possible beliefs and assume, without loss of generality, that they are such that \( p_1 < \ldots < p_n \).

Let \( \mu \) denote the decision maker’s subjective probability distribution over the probabilities of illness. Let \( \Delta \) denote the support of \( \mu \), the (finite) set of probabilities of illness that the decision maker considers possible (i.e., the probabilities \( p_j \) for which \( \mu(p_j) > 0 \)). Eqs. (1) and (2) then become:

\[
V^T = \sum_{\Delta} \mu(p_j) \psi(p_j U(H^T_{h})) + (1 - p_j) U(H^T_{h}),
\]

(A1)

\[
V^{NT} = \sum_{\Delta} \mu(p_j) \psi(p_j U(H^{NT}_{h})) + (1 - p_j) U(H^{NT}_{h}).
\]

(A2)

The decision maker decides to treat when \( V^T \geq V^{NT} \).

An important advantage of the smooth ambiguity model is that it permits application of the machinery of expected utility. Hence, similar to the Arrow–Pratt definition of risk aversion, we can define decision maker 2 to be more ambiguity averse than decision maker 1 if \( -\psi_2^c/\psi_2 > -\psi_1^c/\psi_1 \).

Result 1. Suppose that decision maker 2 is more ambiguity averse than decision maker 1. Suppose also that there is diagnostic ambiguity and that the two decision makers share the same beliefs \( \mu \).
Then decision maker 2 is more inclined to choose treatment than decision maker 1 in the sense that

i. If decision maker 1 decides to treat a patient, decision maker 2 will also decide to treat this patient.

ii. If decision maker 1 is indifferent between treating and not treating a patient, decision maker 2 will decide to treat this patient.

iii. Assuming that there is a probability of illness $p_1$ at which both decision makers decide not to treat and a probability of illness $p_2$ at which both decision makers decide to treat, there is a probability of illness $\hat{p}$ at which decision maker 2 decides to treat and decision maker 1 decides not to treat.

Proof of Result 1. $\phi(EU_{p1}^T) - \phi(EU_{p1}^{NT})$ is negative for $p = 0$, positive for $p = 1$, and increases in $p$. Because $U$ and $\phi$ are differentiable, they are continuous. By the continuity of $U$ and $\phi$, there exists a probability of illness $\hat{p}$ such that $\phi(EU_{p1}^T) = \phi(EU_{p1}^{NT}) = \hat{\phi}$. Let $k$ be an increasing and concave function. By the intermediate value theorem for derivatives (Apostol, 1974, Theorem 5.16), for each $p$ in $\Delta$ there exists a real number $c$ between $\phi(EU_{p1}^T)$ and $\phi(EU_{p1}^{NT})$ such that $k(\phi(EU_{p1}^T)) - k(\phi(EU_{p1}^{NT})) = k(c)(\phi(EU_{p1}^T) - \phi(EU_{p1}^{NT}))$. For $p < \hat{p}$, $\phi(EU_{p1}^T) < \phi(EU_{p1}^{NT})$ and thus, by the concavity of $k$, $k(\phi(EU_{p1}^T)) \geq k(c)\phi(EU_{p1}^T)$ and $k(\phi(EU_{p1}^{NT})) \geq k(c)\phi(EU_{p1}^{NT})$. Because $\phi(EU_{p1}^T) - \phi(EU_{p1}^{NT})$ is negative for $p < \hat{p}$, it follows that $k(\phi(EU_{p1}^T)) - k(\phi(EU_{p1}^{NT})) \geq k(\phi(EU_{p1}^T)) - k(\phi(EU_{p1}^{NT}))$. For $p > \hat{p}$, $k(\phi(EU_{p1}^T)) \leq k(c)$ and $k(\phi(EU_{p1}^{NT})) \leq k(c)$. Because $\phi(EU_{p1}^T) - \phi(EU_{p1}^{NT})$ is positive for $p > \hat{p}$, it follows that $k(\phi(EU_{p1}^T)) - k(\phi(EU_{p1}^{NT})) \leq k(c)$ and $k(\phi(EU_{p1}^{NT})) - k(c)$. Hence, for all $p$, $k(\phi(EU_{p1}^T)) - k(\phi(EU_{p1}^{NT})) \geq k(c)(\phi(EU_{p1}^T) - \phi(EU_{p1}^{NT}))$. It is also true by the concavity of $k$ that for $p < \hat{p}$, $k(\phi(EU_{p1}^T)) \leq k(\phi(EU_{p1}^{NT}))$ and for $p > \hat{p}$, $k(\phi(EU_{p1}^T)) \geq k(\phi(EU_{p1}^{NT}))$. Hence, for all $p$, $k(\phi(EU_{p1}^T)) - k(\phi(EU_{p1}^{NT})) \geq k(c)(\phi(EU_{p1}^T) - \phi(EU_{p1}^{NT}))$. Consequently,

$$\sum_{p1} \mu(p_1)(\phi(EU_{p1}^T) - \phi(EU_{p1}^{NT})) \geq \hat{\phi} \sum_{p1} \mu(p_1)(\phi(EU_{p1}^T) - \phi(EU_{p1}^{NT})) = \phi(EU_{p1}^{NT}) - \phi(EU_{p1}^T).$$

(A3)

Because $k$ is increasing, $k > 0$ and it follows from (A1) that if $\sum_{p1} \mu(p_1)(\phi(EU_{p1}^T) - \phi(EU_{p1}^{NT}))$ is positive then $\sum_{p1} \mu(p_1)(\phi(EU_{p1}^T) - \phi(EU_{p1}^{NT}))$ is also positive. Consequently, if a decision maker whose ambiguity aversion is captured by $\phi$ chooses treatment (i.e. $\sum_{p1} \mu(p_1)(\phi(EU_{p1}^T) - \phi(EU_{p1}^{NT}))$ is positive), a more ambiguity averse decision maker whose ambiguity aversion is captured by an increasing and concave transformation of $\phi$ will also choose treatment. This proves Statements i and ii. Statement iii follows from the continuity of $U$. □

A2. Diagnostic ambiguity with continuous treatment

In Section 2, the decision maker could only choose between treatment and no treatment. This case is not always realistic as in many real-life situations a decision maker can also choose the intensity of treatment. For example, a doctor not only decides on whether to prescribe medication, but also on the appropriate dose. Then the choice variable is continuous instead of dichotomous. We will now show that our conclusion that an increase in ambiguity aversion generates an increase in the propensity to treat is unaffected when the decision maker can select the intensity of treatment. The proof that an increase in ambiguity aversion reduces the propensity to treat in the case of therapeutic ambiguity aversion is largely similar and we will not present it separately.

The treatment variable $t$ is continuous and bounded. As before, there are two states of the world, sick and healthy. The levels of health reached in the two states depend on the selected intensity of treatment and we denote them as $H_{1}(t)$ and $H_{0}(t)$, respectively. We assume that treatment is beneficial when the patient is sick, $H_{1} \geq 0$, and detrimental when the patient is healthy, $H_{0} \leq 0$. However, for any treatment intensity, the patient is always in better health when healthy than when sick: for all $t$, $H_{1}(t) \leq H_{0}(t)$. We assume that the marginal effects of treatment are decreasing both when healthy and when sick: $H_{1}' \leq 0$ and $H_{0}' \leq 0$.

As in Appendix A.1, the number of probabilities of illness that the decision maker considers possible can be any finite number and his beliefs regarding the probability of illness are captured through the function $\mu$.

The decision problem now becomes:

$$\max_{t} \sum_{p} \mu(p) (p_{1}U(H_{1}(t)) + (1 - p_{1})U(H_{0}(t))).$$

(A4)

Result 2. Suppose that decision maker 2 is more ambiguity averse than decision maker 1. Suppose also that there is diagnostic ambiguity and the two decision makers share the same beliefs. Then the optimal level of treatment chosen by decision maker 2 will be at least as large as the optimal level of treatment chosen by decision maker 1.

Proof of Result 2. Denote $EU_{p1}^T = p_{1}U(H_{1}(t)) + (1 - p_{1})U(H_{0}(t))$. The first order condition can be written as:

$$\sum_{p} \mu(p) \phi(EU_{p1}^T) \frac{U'(H_{1}(t))}{EU_{p1}^T} H_{1}(t) + (1 - p_{1})U'(H_{0}(t))H_{0}(t)] = \sum_{p} \mu(p) \phi(EU_{p1}^T)EU_{p1}^T = 0$$

(A5)

$EU_{p1}^T$ can be written as

$$U'(H_{1}(t))H_{1}(t) + p_{1}U'(H_{0}(t))H_{0}(t) - U'(H_{0}(t))H_{1}(t).$$

(A6)

Because $H_{1}(t) \geq 0$ and $H_{0}(t) \leq 0$, it follows that $EU_{p1}^T$ is increasing in $p_{1}$. Because $U' > 0$ and $H_{1}(t) \geq 0$, it follows that $EU_{p1}^T \geq 0$. Because $U' > 0$ and $H_{0}(t) \leq 0$, $EU_{p1}^T \leq 0$. On the other hand, $EU_{p1}^T$ is decreasing in $p_{1}$ by the assumption that for all treatment levels $t$, $H_{1} \leq H_{0}$. Because $\phi' > 0$, it follows from (A5), $EU_{p1}^T$ is increasing in $p_{1}$, and $EU_{p1}^T \geq 0 \geq EU_{p1}^T$ that for each $t$ there must exist a $\hat{p}_{1}$ in $\Delta$ such that $EU_{p1}^T \leq 0$ for those $p_{1} \in \Delta$ for which $p_{1} \leq \hat{p}_{1}$. The $\hat{p}_{1}$ will depend on $t$, but to keep the notation manageable, we will suppress this dependence in what follows. Because $EU_{p1}^T$ is decreasing in $p_{1}$, the $\hat{p}_{1}$ must be true for each treatment level $t$ that for all $p_{1} \in \Delta$.

$$EU_{p1}^T EU_{p1}^T \leq EU_{p1}^{T} p_{1}.$$  

(A7)

If $p_{1} < \hat{p}_{1}$, then $EU_{p1}^T > EU_{p1}^T$ and (A7) follows from $EU_{p1}^T \leq 0$. If $p_{1} > \hat{p}_{1}$, then $EU_{p1}^T \leq EU_{p1}^{T}$ and (A7) follows from $EU_{p1}^{T} > 0$. Let $\phi_{1} = k(\phi_{1})$ with $k$ increasing ($k > 0$) and concave ($k' < 0$) so that decision maker 2 is more ambiguity averse than decision maker 1. Let $t_{1}^*$ be the optimal treatment level of decision maker 1. Because $\phi_{1}$ is increasing, it follows from (A7) that for all $p_{1} \leq \Delta$, $\phi_{1}(EU_{p1}^T)EU_{p1}^T \leq \phi_{1}(EU_{p1}^{T})EU_{p1}^{T}$, and because $k$ is concave, for all $p_{1} \in \Delta$,

$$k(\phi_{1}(EU_{p1}^T))EU_{p1}^T \leq k(\phi_{1}(EU_{p1}^{T}))EU_{p1}^{T}.$$  

(A8)
If \( p_1 < \hat{p} \) then \( EU^*_{\mu_1} > EU^*_{\hat{p}} \), hence, \( \phi_1(EU^*_{\mu_1}) > \phi_1(EU^*_{\hat{p}}) \), hence \( k'(\phi_1(EU^*_{\mu_1})) < k'(\phi_1(EU^*_{\hat{p}})) \) and (A8) follows from \( EU^*_{\mu_1} \leq 0 \).

If \( p_j > \hat{p} \) then \( EU^*_{p_j} < EU^*_{\hat{p}} \), hence, \( \phi_1(EU^*_{p_j}) < \phi_1(EU^*_{\hat{p}}) \), hence \( k'(\phi_1(EU^*_{p_j})) > k'(\phi_1(EU^*_{\hat{p}})) \) and (A8) follows from \( EU^*_{p_j} > 0 \).

Because \( \phi_2 = k'(\phi_1\phi_1') > 0 \), multiplying both sides of (A8) by \( \mu(p_j)\phi_1(EU^*_{p_j}) \) and summing over all \( p_j \in \Delta \) yields

\[
\sum_{p_j} \mu(p_j)\phi_1(EU^*_{p_j})EU^*_{p_j} \leq k'(\phi_1(EU^*_{\hat{p}}))\sum_{p_j} \mu(p_j)\phi_1(EU^*_{p_j})EU^*_{p_j}.
\]

(A9)

Because \( \sum_{p_j} \mu(p_j)\phi_1(EU^*_{p_j})EU^*_{p_j} = 0 \) by the first order condition, it follows that

\[
\sum_{p_j} \mu(p_j)\phi_1(EU^*_{p_j})EU^*_{p_j} \geq 0.
\]

(A10)

Hence, at \( t^*_1 \) the marginal benefits of treatment are positive for decision maker 2 and thus he will increase the level of treatment. Thus \( t^*_1 \geq t^*_1 \), which is the desired result. \( \square \)

A.3. Therapeutic ambiguity with more than two beliefs

In this appendix we show that the conclusion that therapeutic ambiguity averse makes the decision maker less prone to choose treatment holds for any (finite) set of beliefs \( \Delta = \{p_1, \ldots, p_n\} \). We assume without loss of generality that \( p_1 < p_2 < \cdots < p_n \).

Result 3. Suppose that decision maker 2 is more ambiguity averse than decision maker 1. Suppose also that there is therapeutic ambiguity and that the two decision makers share the same beliefs. Then decision maker 2 is less inclined to choose treatment than decision maker 1 in the sense that

i. if decision maker 1 decides not to treat a patient, decision maker 2 will not treat this patient either.

ii. if decision maker 1 is indifferent between treating and not treating a patient, decision maker 2 will decide not to treat this patient.

iii. Assuming that there is a failure rate \( p_1 \) at which both decision makers decide to treat and a failure rate \( p_2 \) at which both decision makers decide not to treat, there is a failure rate \( \hat{p} \) at which decision maker 1 decides to treat and decision maker 2 decides not to treat.

Proof of Result 3. The proof of Result 3 is similar to that of Result 1. Treatment will be chosen if \( \sum_{p_j} \mu(p_j)\phi_1(EU^*_{p_j}) \geq \phi_1(U(H_1^{NT})) \).

\( \phi_1(EU^*) - \phi_1(U(H_1^{NT})) \) is negative for \( p = 0 \), positive for \( p = 1 \), and increases in \( p \). By the continuity of \( U \), there exists a failure rate \( \hat{p} \) such that \( \phi_1(EU^*) = \phi_1(U(H_1^{NT})) = \hat{\phi} \). Let \( k \) be an increasing and concave function. By the intermediate value theorem, for each \( p \) in \( \Delta \) there exists a real number \( c \) between \( \phi_1(EU^*) \) and \( \phi_1(U(H_1^{NT})) \) such that \( k'(\phi_1(EU^*)) - k'(\phi_1(U(H_1^{NT}))) = k'(c) = k'(\psi(\psi^*_1 - \psi(U(H_1^{NT})))) \).

For \( p \leq \hat{p} \), \( \phi_1(EU^*) \geq \phi_1(U(H_1^{NT})) \) and thus, by the concavity of \( k \), \( k'(\phi_1(EU^*)) \leq k'(c) \). Because \( \phi_1(EU^*) - \phi_1(U(H_1^{NT})) \) is positive for \( p \leq \hat{p} \), it follows that \( k'(\phi_1(EU^*)) \leq k'(\phi_1(U(H_1^{NT}))) \).

For \( p > \hat{p} \), \( k'(\phi_1(EU^*)) \geq k'(c) \) and because \( \phi_1(EU^*) \) is decreasing in \( \phi \), it follows that \( k'(\phi_1(EU^*)) \leq k'(\phi_1(U(H_1^{NT}))) \).

Hence, for all \( p \), \( k'(\phi_1(EU^*)) \leq k'(\phi_1(U(H_1^{NT}))) \).

It is also true, by the concavity of \( k \), that for \( p < \hat{p} \), \( k'(\phi_1(EU^*)) \leq k'(\phi_1(U(H_1^{NT}))) \) and for \( p > \hat{p} \), \( k'(\phi_1(EU^*)) \geq k'(\phi_1(U(H_1^{NT}))) \). Hence, for all \( p \), \( k'(\phi_1(EU^*)) \leq k'(\phi_1(U(H_1^{NT}))) \) and (A8) follows from \( EU^*_{p_j} > 0 \).

Because \( \phi_2 = k'(\phi_1\phi_1') > 0 \), multiplying both sides of (A8) by \( \mu(p_j)\phi_1(EU^*_{p_j}) \) and summing over all \( p_j \in \Delta \) yields

\[
\sum_{p_j} \mu(p_j)\phi_1(EU^*_{p_j})EU^*_{p_j} \leq k'(\phi_1(EU^*_{\hat{p}}))\sum_{p_j} \mu(p_j)\phi_1(EU^*_{p_j})EU^*_{p_j}.
\]

(A9)

Because \( \sum_{p_j} \mu(p_j)\phi_1(EU^*_{p_j})EU^*_{p_j} = 0 \) by the first order condition, it follows that

\[
\sum_{p_j} \mu(p_j)\phi_1(EU^*_{p_j})EU^*_{p_j} \geq 0.
\]

(A10)

Hence, at \( t^*_1 \) the marginal benefits of treatment are positive for decision maker 2 and thus he will increase the level of treatment. Thus \( t^*_1 \geq t^*_1 \), which is the desired result. \( \square \)

A.4. Results under prospect theory

In this appendix, we will show that the conclusions derived under the smooth model also hold under prospect theory. Start with the case of diagnostic ambiguity. Let \( E(\hat{p}) \) denote the expected value of the probabilities of illness that the decision maker considers possible (i.e., the expectation of the \( p \) in \( \Delta \)). Under prospect theory, treatment and no treatment are evaluated as

\[
V^T = W(\Delta)U(H_1^T) + (1 - W(\Delta))U(H_0^T) = m(E(\hat{p}))(U(H_1^T) + (1 - m(E(\hat{p})))U(H_0^T).
\]

The decision maker is ambiguity neutral if \( m(E(\hat{p})) = m(E(\hat{p})) \) for all \( E(\hat{p}) \), which implies that \( m \) is linear. Ambiguity aversion corresponds with \( m \) concave (Baillon et al., 2012a). Concavity of \( m \) means that the decision maker overweightings the higher probabilities of illness, i.e., he behaves in a pessimistic manner. Extrem ambiguity aversion means that he only considers \( p_1 \) the highest probability of illness.

Fig. 1A illustrates the impact of diagnostic ambiguity aversion under prospect theory. Panel A shows that under ambiguity neutrality, \( PT_0[p]^{T} \) the prospect theory value of treatment [no treatment] when \( E(\hat{p}) = p \) is linear in \( p \). The slope of \( PT_0^{T} \) is steeper than the slope of \( PT_0^{T} \) because the spread of the outcomes of no treatment exceeds the spread of the outcomes of treatment. The decision maker is indifferent between treatment and no treatment for \( E(\hat{p}) = \hat{p} \).

Panel B shows that under ambiguity aversion (\( m \) concave), the relationship between the prospect theory value and \( E(\hat{p}) \) becomes convex. This makes treatment more attractive and the indiscrimination value falls from \( \hat{p} \) to \( \bar{p} \). Intuitively, ambiguity aversion has the effect of putting extra weight on the higher probabilities of illness. This favors the treatment option and, thus, generates an increase in the propensity to treat.

The case of therapeutic ambiguity is similar. We have

\[
V^T = W(\Delta)U(H_1^{T-}) + (1 - W(\Delta))U(H_0^{T-}) = m(E(\hat{p}))(U(H_1^{T-}) + (1 - m(E(\hat{p})))U(H_0^{T-})),
\]

(A14)

\[
V^{NT} = U(H_1^{NT}).
\]

(A15)

\( ^8 \) Often the prospect theory formulas are presented in a dual way with the weight \( W(\Delta) \) applied to the best outcome. This formulation is equivalent to the one we use here but it is more cumbersome in terms of the notation used in this paper.

\( ^9 \) Prospect theory also assumes that the decision maker weights unambiguous probabilities to reflect his attitudes towards risk. We will, without loss of generality, abstract from this to study the pure effect of ambiguity aversion.
**Fig. A1.** Panel A shows that under diagnostic ambiguity neutrality, the prospect theory value of no treatment ($PTNT_p$) and of treatment ($PT^*_p$) decreases linearly with the expected probability of illness $E(\hat{p})$. For probability $\hat{p}$, the ambiguity-neutral decision maker is indifferent between treatment and no treatment. Panel B shows that under ambiguity aversion, $PT_{NT}^T$ and $PT^T_p$ become convex functions of $E(\hat{p})$ and the indifference probability shifts to $\hat{p}$, implying a higher propensity to treat.

**Fig. A2.** Ambiguity aversion reduces the propensity to treat in the case of therapeutic ambiguity. Ambiguity aversion shifts the line displaying the value of treatment under prospect theory downwards, making it a convex function of the expected probability of treatment failure ($E(\hat{p})$). This makes no treatment more attractive and the indifference failure rate of treatment shifts from $\hat{p}$ to $\hat{p}$.

Ambiguity aversion increases the concavity of $m(E(\hat{p}))$ and, hence, the weight assigned to the outcome of treatment failure. This, in turn, decreases the attractiveness of treatment. Fig. A2 illustrates that the curve depicting the value of treatment under prospect theory is convex in $E(\hat{p})$ and shifts downwards due to ambiguity aversion. The new expected failure rate that is acceptable to the decision maker shifts to $\hat{p}$. Hence, ambiguity aversion decreases the propensity to treat.

**References**


