The Value of a Statistical Life Under Changes in Ambiguity

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February 2017

Abstract

The value of a statistical life (VSL) is widely used in policy evaluation. Most policy decisions are made under ambiguity. This paper studies the effect of changes in ambiguity perception on the value of a statistical life (VSL). We propose a definition of increases in ambiguity perception based on Ekern's definition of increases in risk. Ambiguity aversion alone is not sufficient to lead to an increase in VSL when the decision maker perceives more ambiguity. Our results highlight the importance of higher order ambiguity attitudes, particularly ambiguity prudence.

KEYWORDS: Value of a statistical life, ambiguity, prudence, smooth ambiguity model, neo-additive preferences.

JEL classification: D81, I18, Q51

Acknowledgements: We are grateful to Aurélien Baillon for his comments on an earlier version of this paper.

1. Introduction

Policy interventions often involve changes in the risks to human life. Examples include measures to mitigate the effects of climate change, investments in road safety, and the reimbursement of new medical treatments. The most common metric to assess the effectiveness of these interventions is the value of a statistical life (VSL). The VSL is calculated by asking people for their willingness to trade-off wealth against reductions in mortality risk.

A large literature has explored how the VSL changes with, amongst others, wealth (Hammitt 2000), the baseline mortality risk (Pratt and Zeckhauser 1996), background financial risk (Eeckhoudt and Hammitt, 2001), age (Viscusi and Aldy, 2003), health status (Hammitt, 2002), and altruism (Andersson and Lindberg, 2009). All these studies take the mortality risk (and changes therein) as known. However, in most real-life decisions mortality risks are not precisely known and subject to ambiguity. For example, experts have conflicting opinions about the exact risks to human life of climate change. Most experts agree that it exists, but they disagree about its exact size leading to ambiguous estimates.

Ellsberg's famous paradox (1961) showed that ambiguity affects people's behavior. Ambiguity aversion can account for several empirical puzzles that tradition economics has difficulty to explain. Examples are the stock market participation puzzle (the finding that many people do not buy stocks even though it is the rational thing to do according to standard portfolio theory[1]), home bias (the finding that people invest too much in stocks

[1] See e.g. Mankiw and Zeldes (1991) and Haliassos and Bertaut (1995)

from their own country[2]), and the low take-up of freely available genetic tests (Hoy et al. 2014). Ellsberg's paradox led to a variety of new ambiguity theories (Ryan 2009, Gilboa and Marinacci 2013, Machina and Siniscalchi 2014). Of these models, the smooth model of Klibanoff, Marinachi, and Mukerji (KMM) (2005) is most widely used in economic applications.

Treich (2010) explored the impact of ambiguity on VSL under the smooth model. He showed that under plausible assumptions ambiguity leads to an increase in the VSL of an ambiguity averse decision maker compared with the situation in which there is no ambiguity.

This paper extends Treich (2010) to the case where both decision alternatives are ambiguous, but differ in their degree of ambiguity. This case is arguably more realistic as there are few real-world decisions where probabilities are exactly known, but it is often possible to distinguish degrees of ambiguity. For example, more research into climate change may lead to more precise estimates of the risks to human life even when the exact risks remain unknown. Treich's analysis might suggest that an ambiguity averse decision maker will always have a higher VSL in decisions with more ambiguity. Our analysis shows that this conjecture is not true. The effect of general changes in perceived ambiguity is more complex than the comparison between ambiguity and no ambiguity. It requires information about higher order ambiguity preferences than ambiguity aversion alone. In particular, our results show the importance of ambiguity prudence.

The paper is structured as follows. In Section 2 we give background and derive the VSL under risk. Section 3 then introduces ambiguity and derives the VSL under KMM's smooth

[2] French and Poterba (1991)

model. Section 4 considers mean-preserving spreads in ambiguity perception. We show that Treich's result cannot be generalized to general mean–preserving spreads in ambiguity perception and we highlight the important role that ambiguity prudence plays in this context. Section 5 considers higher order changes in ambiguity. We define the concept of n^{th} order increases in ambiguity and show that the effect of these increases on VSL depends on higher order ambiguity preferences under the smooth model.

In Section 6, we consider the case of ambiguity seeking. Ellsberg (1961) already pointed out that in some situations people may be ambiguity seeking. The empirical literature has shown that this particularly happens for unlikely events and when losses are involved (Trautmann and van de Kuilen 2016; Wakker 2010). As the risk of death is typically small, studying the effect of ambiguity on the VSL for ambiguity seekers is clearly relevant.

Like Treich (2010), most of our analysis uses KMM's smooth model. The smooth model incorporates ambiguity aversion using a different utility function for uncertainty than for risk. A different strand of the ambiguity literature models ambiguity aversion through the weighting of events. The main models in this class are Choquet expected utility (Schmeidler 1989) and the multiple priors models (Gilboa and Schmeidler 1989). In Section 7, we extend our analysis to Chateauneuf et al.'s (2007) neo-additive preferences, which have intuitive interpretations in terms of Choquet expected utility and in terms of multiple priors. We show that our conclusions remain valid under neo-additive preferences: ambiguity aversion alone does not necessarily lead to an increase in the VSL when perceived ambiguity increases, but higher order preferences, in particular ambiguity prudence, play a key role.

2. Background

The standard VSL model assumes that a decision maker (DM) evaluates decisions involving a fatality risk by (state-dependent) expected utility:

$$V_0 = (1-p)U_l(w) + pU_d(w). \tag{1}$$

In Eq. (1), p is the probability that the DM dies during the current period, $U_l(w)$ is the DM's utility of wealth if he survives the period and $U_d(w)$ is his utility of wealth if he does not survive (i.e. his utility of a bequest). If the DM has no bequest motive then $U_d(w)$ is zero for all wealth levels.

It is common to assume that the DM prefers more wealth to less ($U_l'(w) > 0$ and $U_d'(w) > 0$ for all w), that he is risk averse ($U_l''(w) < 0$ and $U_d''(w) < 0$ for all w), and that both the utility of wealth and the marginal utility of wealth are always higher when alive than when dead ($U_l(w) > U_d(w)$, $U_l'(w) > U_d'(w)$ for all w). The VSL is the marginal rate of substitution between wealth and mortality risk. It is obtained by totally differentiating Eq. (1) with respect to p and w holding expected utility constant:

$$VSL_r = \frac{dw}{dp} = \frac{U_l(w) - U_d(w)}{(1-p)U_l'(w) + pU_d'(w)} \tag{2}$$

The subscript r serves as a reminder that we are considering the case of risk where the mortality risk is objectively known. Under the assumptions made, VSL_r is strictly positive and increases with wealth and the mortality risk p. The positive relation between VSL and the mortality risk has been coined the "dead anyway" effect by Pratt and Zeckhauser (1996). It expresses that, abstracting from bequest considerations, a DM who faces a high probability of death will be inclined to spend as much as he can on mortality risk reduction as he is unlikely to survive anyway.

3. The smooth ambiguity model

We will now explore the impact of ambiguity. Suppose the baseline mortality risk p is no longer objectively known, but is ambiguous. We express this by adding a random variable $\tilde{\varepsilon}$, which reflects the DM's perceived ambiguity, to p so that the new mortality risk becomes: $\tilde{p} = p + \tilde{\varepsilon}$. To avoid negative probabilities or probabilities larger than 1, the support of $\tilde{\varepsilon}$ is restricted to $[-p, 1-p]$. We assume that the DM behaves according to the smooth model of KMM (2005) according to which he evaluates the mortality risk as:

$$W_{\tilde{\varepsilon}} = \varphi^{-1}(E(\varphi((1-\tilde{p})U_l(w) + \tilde{p}U_d(w)))). \quad (3)$$

In the smooth model the DM's ambiguity perception is modeled by a second order distribution $\tilde{p}$, which reflects his beliefs about the mortality risk. The function φ is increasing and reflects the DM's ambiguity attitudes. If φ is everywhere concave ($\varphi'' < 0$) then the DM is (uniformly) ambiguity averse. If φ is everywhere convex ($\varphi'' > 0$) then the DM is (uniformly) ambiguity seeking, If φ is linear ($\varphi'' = 0$), the DM is ambiguity neutral and the smooth model is equivalent to subjective expected utility. Then the DM behaves according to Eq.(2) with $p = E(\tilde{p})$. In Eq.(3) we use the subscript $\tilde{\varepsilon}$ to emphasize that utility depends on the DM's ambiguity perception.

To facilitate comparisons, we will assume that $E[\tilde{\varepsilon}] = 0$ and, thus, $E[\tilde{p}] = p$. In other words, ambiguity leads to an increase in the spread of the distribution of mortality risks that the DM perceives as possible, but it does not lead to a systematic bias in the perceived risks. We will discuss the effects of such a systematic bias in Section 7.

We obtain the VSL under the smooth ambiguity model by totally differentiating Eq. (3) with respect to p and w. This gives:

$$VSL_{\tilde{\varepsilon}} = \frac{dw}{dp} = \frac{(U_l(w) - U_d(w))E\left[\varphi'\left((1-\tilde{p})U_l(w) + \tilde{p}U_d(w)\right)\right]}{E\left[\left((1-\tilde{p})U_l'(w) + \tilde{p}U_d'(w)\right)\varphi'\left((1-\tilde{p})U_l(w) + \tilde{p}U_d(w)\right)\right]} \tag{4}$$

Treich (2010) showed that an ambiguity-averse DM will have a higher VSL under ambiguity than under risk: $VSL_{\tilde{\varepsilon}} > VSL_r$. The intuition is that an ambiguity averse DM concentrates on the higher mortality risks and we noticed before that higher mortality risks increase the VSL as a result of the dead anyway effect. Treich's result also implies that the more ambiguity averse the DM is (as reflected by a more concave φ in the smooth model), the higher is his $VSL_{\tilde{\varepsilon}}$.

4. Changes in perceived ambiguity

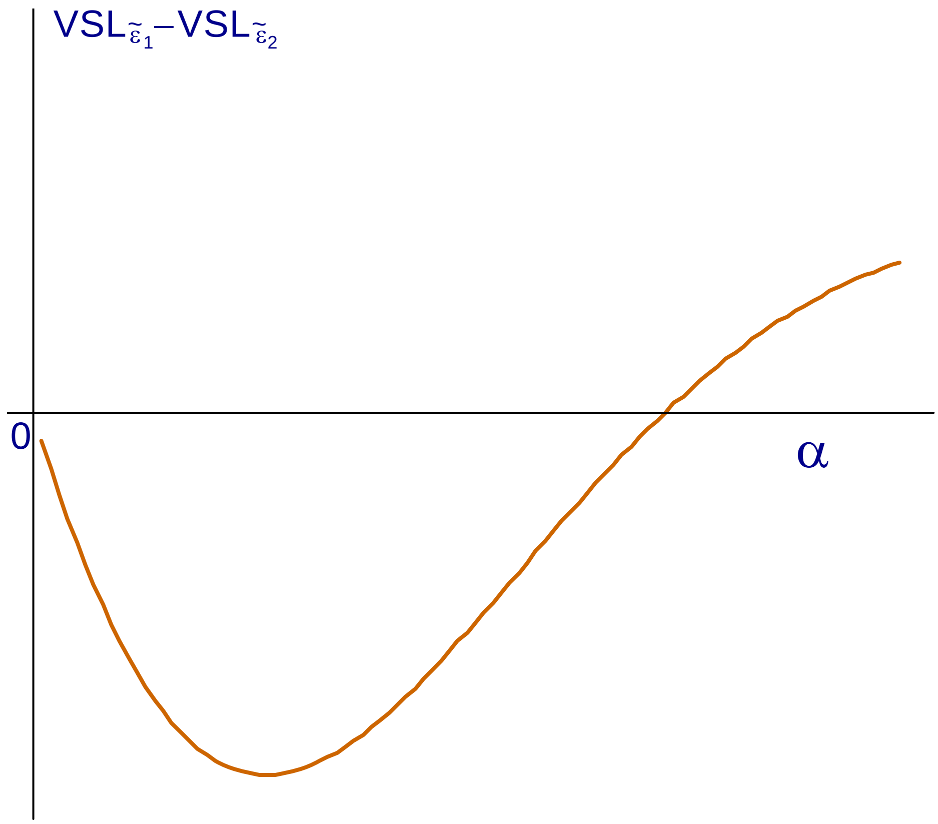
Consider two situations for which the DM's levels of perceived ambiguity are described by the random variables $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$. If we write $\widetilde{p_\iota} = p + \tilde{\varepsilon}_i, i = 1,2$, Eq. (4) becomes:

$$VSL_{\widetilde{\varepsilon_\iota}} = \frac{dw}{dp} = \frac{(U_l(w) - U_d(w))E\left[\varphi'\left((1-\widetilde{p_\iota})U_l(w) + \widetilde{p_\iota}\, U_d(w)\right)\right]}{E\left[(1-\widetilde{p_\iota})U_l'(w) + \widetilde{p_\iota}U_d'(w)\varphi'\left((1-\widetilde{p_\iota})U_l(w) + \widetilde{p_\iota}U_d(w)\right)\right]} \tag{5}$$

First consider the case where $\tilde{\varepsilon}_2$ is a mean-preserving spread of $\tilde{\varepsilon}_1$. Higher order changes in ambiguity are studied in the next Section. Because $\tilde{\varepsilon}_2$ is a mean-preserving spread of $\tilde{\varepsilon}_1$, $E[\tilde{\varepsilon}_1] = E[\tilde{\varepsilon}_2]$, but $Var[\tilde{\varepsilon}_1] < Var[\tilde{\varepsilon}_2]$. Treich (2010) showed that if $\tilde{\varepsilon}_1 = 0$

$VSL_{\tilde{\varepsilon}_2}$ will exceed $VSL_{\tilde{\varepsilon}_1}$. The next example shows that this is no longer true when $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ are both nondegenerate.

Figure 1: The relationship between $VSL_{\tilde{\varepsilon}_1} - VSL_{\tilde{\varepsilon}_2}$ and α in the example.



EXAMPLE:

Let $\varphi(x) = -\frac{e^{-\alpha x}}{\alpha}, \alpha > 0$. We can measure the intensity of the DM's ambiguity aversion as $-\frac{\varphi''}{\varphi'}$, much like the well-known Arrow-Pratt index measures the DM's risk aversion. The function $\varphi(x) = -\frac{e^{-\alpha x}}{\alpha}$ has the attractive property that ambiguity aversion is constant and equal to α. Normalize wealth to 1 and let $U_l = (1 - e^{-.1x})$ and $U_d = 0.5 * (1 - e^{-.05x})$. Then $U_l(x) > U_d(x)$ and $U_l'(x) > U_d'(x)$ for all positive wealth levels x. We write $(p_1: x_1, \dots, p_n: x_n)$ for the random variable that gives x_j with probability $p_j, j = 1, \dots, n$. Consider two random variables $\tilde{\varepsilon}_1 = (0.5: -0.2, 0.5: 0.2)$ and $\tilde{\varepsilon}_2 = (0.125: -0.5, 0.375: -0.1, 0.5: 0.2)$. Then $E[\tilde{\varepsilon}_1] = E[\tilde{\varepsilon}_2] = 0$ and $\tilde{\varepsilon}_2$ is a mean-preserving spread of $\tilde{\varepsilon}_1$. Figure 1 displays the relation between $VSL_{\tilde{\varepsilon}_1} - VSL_{\tilde{\varepsilon}_2}$ and α. The figure shows that initially $VSL_{\tilde{\varepsilon}_2}$ exceeds $VSL_{\tilde{\varepsilon}_1}$, but as α increases the difference becomes smaller and for α sufficiently large the relation is reversed and $VSL_{\tilde{\varepsilon}_2}$ is less than $VSL_{\tilde{\varepsilon}_1}$. Because the intensity of the DM's ambiguity aversion is equal to α, the figure shows that more ambiguity aversion does not necessarily lead to a higher VSL when the DM's ambiguity perception increases.

In other words, our example shows the following:

RESULT 1: Let $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ be two nondegenerate random variables with $\tilde{\varepsilon}_2$ a mean-preserving spread of $\tilde{\varepsilon}_1$. Then the sign of $VSL_{\tilde{\varepsilon}_2} - VSL_{\tilde{\varepsilon}_1}$ is indeterminate.

Why doesn't Treich's result carry over to the more general case where both risks are ambiguous? For notational convenience, let $\tilde{Z}_i = (1 - \tilde{p}_i)U_l(w) + \tilde{p}_i U_d(w)$ and $\tilde{Z}_i' = (1 - \tilde{p}_i)U_l'(w) + \tilde{p}_i U_d'(w), i = 1,2$. In the special case where one situation is unambiguous ($\tilde{\varepsilon}_1 = 0$) $\tilde{Z}_1'$ is constant. Then the denominator of Eq. (5) becomes

$E[\tilde{Z}_1']E[\varphi'(\tilde{Z}_1)]$. Because $\widetilde{\varepsilon_2}$ is a mean-preserving spread of $\tilde{\varepsilon}_1$, $E[\tilde{Z}_1'] = E[\tilde{Z}_2']$. Substituting $E[\tilde{Z}_1'\varphi'(\tilde{Z}_1)] = E[\tilde{Z}_1']E[\varphi'(\tilde{Z}_1)]$ and $E[\tilde{Z}_1'] = E[\tilde{Z}_2']$ into Eq. (5) gives after some rearranging that $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$ iff $E[\tilde{Z}_2']E[\varphi'(\tilde{Z}_2)] > E[\tilde{Z}_2'\varphi'(\tilde{Z}_2)]$. This last inequality holds if the covariance between $\tilde{Z}_2'$ and $\varphi'(\tilde{Z}_2)$ is negative, which is true for an ambiguity averse DM. However, if $\tilde{\varepsilon}_1$ is also ambiguous, $\tilde{Z}_1'$ is no longer constant and the above argument can no longer be applied.

To ensure that a mean-preserving spread in ambiguity always leads to an increase in VSL, we must impose additional conditions on the DM's ambiguity attitudes, in particular ambiguity prudence. Result 2 gives sufficient conditions. Baillon (forthcoming) gave a model-free definition of ambiguity prudence. Ambiguity prudence is a plausible assumption. Baillon (forthcoming) shows that it is implied by most ambiguity models and that it correlates with economic behavior. It reflects the intuition that a decision maker prefers to spread harms across events rather than to concentrate them in one or a few events. In our decision context it means that the DM prefers to combine ambiguity with states of the world in which the mortality risk is low rather than with states of the world in which the mortality risk is high. The importance of prudence in explaining economic behavior is widely-documented for decisions under risk where it corresponds to a preference for precautionary saving (Kimball 1990). For ambiguity, Guerdjikova and Sciubba (2015) show that ambiguity prudence plays a crucial role in the survival of ambiguity averse agents in the market. Baillon (forthcoming) illustrates its importance for prevention behavior. Other recent illustrations are Berger (2014, 2016) on saving and prevention, Gierlinger and Gollier (2015) on the socially efficient discount rate, and Peter and Ying (2017) on insurance.

Under the smooth model, ambiguity prudence is equivalent to $\varphi''' > 0$. We can define an index of ambiguity-prudence as $-\frac{\varphi'''}{\varphi''}$. This index measures the strength of the DM's ambiguity prudence. It reflects the extent to which the DM cares about the skewness of his ambiguity perceptions. If $\varphi''' = 0$, the DM does not care about skewness. If $\varphi''' < 0$, the DM is ambiguity imprudent.

RESULT 2: For all $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ with $\tilde{\varepsilon}_2$ a mean-preserving spread of $\tilde{\varepsilon}_1$,

i. If the DM is ambiguity prudent and $-\frac{\varphi'''}{\varphi''} < 2S^*$ then $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$.

ii. If $\varphi''' = 0$ then also $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$.

iii. If the DM is ambiguity imprudent then the sign of $VSL_{\tilde{\varepsilon}_2} - VSL_{\tilde{\varepsilon}_1}$ is indeterminate.

In Result 2 part (i), $S^* = \frac{U_l' - U_d'}{(U_l - U_d)\left(Z_i' - min(\tilde{\varepsilon}_2)(U_l' - U_d')\right)}$, where min $(\tilde{\varepsilon}_2)$ denotes the minimum value that the random variable $\tilde{\varepsilon}_2$ takes.[3] Result 2 part (ii) says that a mean-preserving spread in the DM's perceived ambiguity always leads to an increase in the VSL if the DM does not care about the skewness of the distribution of his perceived mortality risks. This is the only case where we can straightforwardly extend Treich's result. If the DM is ambiguity prudent then a mean-preserving spread in ambiguity perception also leads to an increase in the VSL if his ambiguity prudence is not too extreme. The reason why ambiguity prudence cannot be too extreme is to exclude cases as the one in our example. For $\varphi(x) = -\frac{e^{-\alpha x}}{\alpha}$ the index of ambiguity prudence is equal to α and the example showed that if ambiguity prudence becomes too large then Result 2 part (i) no longer holds. Intuitively, $\tilde{\varepsilon}_2$ can be more left-skewed than $\tilde{\varepsilon}_1$. An ambiguity prudent likes this negative

[3] min $(\tilde{\varepsilon}_2) \leq$ min $(\tilde{\varepsilon}_1)$ because $\tilde{\varepsilon}_2$ is a mean-preserving spread of $\tilde{\varepsilon}_1$.

skewness. On the other hand, the DM is also ambiguity averse and dislikes the greater ambiguity involved in $\tilde{\varepsilon}_2$. If his ambiguity prudence is not too strong the negative effect of ambiguity aversion will dominate and the increase in ambiguity leads to an increase in VSL. The proof of Result 2 is in the Appendix.

5. Generalization to higher order changes in ambiguity

We will now extend our analysis to more general changes in ambiguity perception. Consider, as before, two random variables $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ with $\tilde{p}_i = p + \tilde{\varepsilon}_i, i = 1{,}2$. Let F_i be the cumulative probability distribution of $\tilde{p}_i$. $[a, b]$ denotes the support of $\tilde{p}_1 \cup \tilde{p}_2$, i.e. the DM believes that mortality risks outside the interval $[a, b]$ are impossible and, thus, $F_i(a) = 0$ and $F_i(b) = 1, i = 1{,}2$. Obviously, $[a, b]$ is a subset of [0,1], as probabilities cannot be negative or exceed 1.

Rewrite $F_i^0 = \tilde{p}_i, F_i^1 = F_i, i = 1{,}2$ and define repeated integrals F_i^k for $k \geq 1$ by:

$$F_i^k(p) = \int_z^x F_i^{k-1}(q)dq. \qquad (6)$$

For decision under risk, Ekern (1980) gave a definition of more n^{th} order risk aversion when risks about wealth are introduced. Caballé and Pomanski (1996) defined a DM as *mixed risk averse* if his von Neumann Morgenstern utility function u has positive odd and negative even derivatives: for all $k = 1, .., n$, $(-1)^{(k+1)}u^{(k)} > 0$. Ekern's (1980) definition implies that a mixed risk averse DM will find any n^{th} order increase in risk undesirable.

A DM is *mixed ambiguity averse* if his ambiguity function φ has positive odd and negative even derivatives: for all $k = 1, .., n$, $(-1)^{(k+1)}\varphi^{(k)} > 0$. Courbage and Rey (2016) defined a change in ambiguity $\tilde{\varepsilon}_2$ as more ambiguous than another change $\tilde{\varepsilon}_1$ if every mixed ambiguity averse DM prefers $\tilde{p}_1 = p + \tilde{\varepsilon}_1$ to $\tilde{p}_2 = p + \tilde{\varepsilon}_2$. Using this definition, we can now formally define what it means to have more n^{th} order ambiguity in the context of our decision problem.

Definition 1: $\tilde{\varepsilon}_2$ has more n^{th} order ambiguity than $\tilde{\varepsilon}_1$, written $\tilde{\varepsilon}_2 \succcurlyeq_n \tilde{\varepsilon}_1$ if

i. $F_2^k(b) = F_1^k(b)$ for $k = 1, \dots, n$

ii. If n is odd, $F_2^n(p) \leq F_1^n(p)$ for all $p \in [a, b]$ and there exists a $p \in [a, b]$ for which $F_2^n(p) < F_1^n(p)$. If n is even, $F_2^n(p) \geq F_1^n(p)$ for all $p \in [a, b]$ and there exists a $p \in [a, b]$ for which $F_2^n(p) > F_1^n(p)$.

Part (i) of Definition 1 implies that the $(n - 1)$ first moments of F_1 and F_2 are equal. Part (ii) implies that the n-th moment of F_2 exceeds the n-th moment of F_1. So if $\tilde{\varepsilon}_2$ has more first order ambiguity than $\tilde{\varepsilon}_1$ then the mean of $\tilde{\varepsilon}_2$ exceeds the mean of $\tilde{\varepsilon}_1$. In other words, $\tilde{\varepsilon}_2$ has a higher expected mortality risk than $\tilde{\varepsilon}_1$. If $\tilde{\varepsilon}_2$ has more second order ambiguity than $\tilde{\varepsilon}_1$ then $\tilde{\varepsilon}_2$ is a mean-preserving spread of $\tilde{\varepsilon}_1$, the case we considered in Section 3. If $\tilde{\varepsilon}_2$ has more third order ambiguity than $\tilde{\varepsilon}_1$ then $\tilde{\varepsilon}_2$ can be obtained from $\tilde{\varepsilon}_1$ by a series of mean-variance-preserving-transformations (Menezes at el., 1980). These transformations do not affect the mean and the variance but transfer ambiguity from lower to higher values of the mortality risk with the result that the distribution becomes more skewed to the right.

Lemma A1 in the Appendix shows that a mixed ambiguity averse DM will indeed always dislike increases in n^{th} order ambiguity. The intuition is that a mixed ambiguity averse DM prefers to distribute harms across states of nature rather than to concentrate them in one state. In other words, he prefers to combine good with bad rather than combining good with good (and bad with bad). An example may clarify. Suppose a DM faces a certain increase k in his perceived mortality risk and a zero mean random variable $\tilde{\varepsilon}$. All ambiguity averse DMs will dislike these two changes and they are both perceived as harmful. A mixed ambiguity averse DM will then prefer the change $[0.5: k; 0.5: \tilde{\varepsilon}\,]$ in his ambiguity perception to the change $[0.5: k + \tilde{\varepsilon}\,; 0.5: 0]$. In the latter change, the two harms are concentrated in one state, whereas in the former change they are divided over the two states. The two changes have the same mean (k) and the same variance $(0.5k^2)$, but $[0.5: k + \tilde{\varepsilon}\,; 0.5: 0]$ is more skewed to the right. Definition 1 reflects that a mixed ambiguity averse DM dislikes positive skewness.

The assumption of mixed ambiguity aversion is plausible and common in the literature. Brockett and Golden (1987) have pointed out that for all commonly used functions in economic theory with a positive first derivative and a negative second derivative, each successive derivatives change sign. Thus, in our case all functions φ that reflect ambiguity aversion must be mixed ambiguity averse. Examples of functions that are mixed ambiguity averse are the constant ambiguity aversion function $\varphi(x) = -e^{-\alpha x}/\alpha$, for $\alpha > 0$, which we used in our example and which was also used by KMM (2005), and the functions used in Ju and Miao (2012) and in Gollier (2011).

We can now state the generalization of Result 2 to n^{th}order changes in ambiguity.

Result 3. Let the DM be mixed ambiguity averse. For all $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ such that $\tilde{\varepsilon}_2 \succcurlyeq_n \tilde{\varepsilon}_1$, if $-\frac{\varphi^{(n+1)}}{\varphi^{(n)}} < nS^*$ then $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$.

S^* is defined as $S^* = \frac{U_l' - U_d'}{(U_l - U_d)\left(Z_i' - \min\{\tilde{\varepsilon}_1, \tilde{\varepsilon}_2\}(U_l' - U_d')\right)}$. In words, Result 3 says that if the DM's n^{th} order ambiguity aversion is not too extreme, n^{th} order increases in ambiguity will lead to an increase in his VSL.

6. Ambiguity seeking

Thus far we have only analyzed the preferences of an ambiguity averse DM. However, empirical evidence suggests that uniform ambiguity aversion is rare and that there are decision contexts in which ambiguity seeking prevails (Trautmann and van de Kuilen 2016; Wakker 2010).

An ambiguity-seeking DM is characterized by $\varphi' > 0$ and $\varphi'' > 0$. The introduction of ambiguity increases the utility of an ambiguity seeker. Result 4 shows that the introduction of ambiguity lowers the VSL of an ambiguity seeking DM compared with the situation with no ambiguity:

RESULT 4: Let the DM be ambiguity seeking. Let $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ be two random variables with $\tilde{\varepsilon}_1 = 0$ and $\tilde{\varepsilon}_2$ a mean-preserving spread of $\tilde{\varepsilon}_1$. Then $VSL_{\tilde{\varepsilon}_2} < VSL_{\tilde{\varepsilon}_1}$.

Result 5 summarizes what happens if $\tilde{\varepsilon}_2$ is a mean-preserving spread of $\tilde{\varepsilon}_1$ and both situations are ambiguous (i.e., both $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ are nondegenerate). It is the immediate counterpart of Result 2.

Result 5: Let the DM be ambiguity seeking. For all $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ with $\tilde{\varepsilon}_2$ a mean-preserving spread of $\tilde{\varepsilon}_1$,

i. If the DM is ambiguity imprudent and $-\frac{\varphi'''}{\varphi''} < 2S^*$ then $VSL_{\tilde{\varepsilon}_2} < VSL_{\tilde{\varepsilon}_1}$.

ii. If $\varphi''' = 0$ then also $VSL_{\tilde{\varepsilon}_2} < VSL_{\tilde{\varepsilon}_1}$.

iii. If the DM is ambiguity prudent then the sign of $VSL_{\tilde{\varepsilon}_2} - VSL_{\tilde{\varepsilon}_1}$ is indeterminate.

In part (i), $S^* = \frac{U_l' - U_d'}{(U_l - U_d)\left(Z_i' - \min(\tilde{\varepsilon}_2)(U_l' - U_d')\right)}$ as in Result 2.

Baillon (forthcoming) points out that most ambiguity models imply ambiguity prudence. Hence, the message of Result 5 is that for most ambiguity models we cannot sign the effect of an increase in ambiguity for an ambiguity seeking DM.

Finally, Result 6 summarizes what happens in the case of higher order increases in ambiguity. While Results 4 and 5 are rather straightforward counterparts of Results 1 and 2, Result 6 is a bit different. The intuition underlying this difference is that, unlike for an ambiguity averse DM, for an ambiguity seeking DM the first and second derivative of φ do not change sign.

Result 6. Let the DM be ambiguity seeking. For all $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ such that $\tilde{\varepsilon}_2 \succcurlyeq_n \tilde{\varepsilon}_1$ with $n \geq 3$:

i. If $\varphi^{(n)} > 0$ and $\varphi^{(n+1)} < 0$ for n odd and if $-\frac{\varphi^{(n+1)}}{\varphi^{(n)}} < nS^*$ then $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$.

ii. If $\varphi^{(n)} > 0$ and $\varphi^{(n+1)} < 0$ for n even and if $-\frac{\varphi^{(n+1)}}{\varphi^{(n)}} < nS^*$ then $VSL_{\tilde{\varepsilon}_2} < VSL_{\tilde{\varepsilon}_1}$.

with $S^* = \frac{U_l' - U_d'}{(U_l - U_d)\left(Z_i' - \min\{\tilde{\varepsilon}_1, \tilde{\varepsilon}_2\}(U_l' - U_d')\right)}$.

In words, we get conclusive results for an ambiguity seeker if the higher order derivatives change sign and if the ratio of the derivatives is not too extreme. In all other cases no conclusive results can be derived. Analogously to the case of mixed risk seeking (Crainich et al. 2013), we say that a DM is *mixed ambiguity seeking* if the derivatives of his ambiguity function φ are always positive: $\varphi^{(n)} > 0$ for all n. Mixed ambiguity seeking can be explained by a preference for combining good with good. Result 6 shows that we can make no clear predictions about the effect of n-th order increases in ambiguity for mixed ambiguity seekers.

7. Neo-additive preferences

The empirical literature gives no clear answer which ambiguity model best describes people's preferences. While the results in Cubitt et al. (2016) are consistent with the smooth model, Baillon and Bleichrodt (2015) and Chew et al. (2016) observed that models like Choquet expected utility (Schmeidler 1989) and α-maxmin (Ghirardato et al. 2004) could better explain their data. In this Section, we explore the robustness of our results under the neo-additive model of Chateauneuf et al. (2007). Neo-additive preferences are a special case of Choquet expected utility and they also have an interpretation in terms of α-maxmin (see for instance Baillon et al. (forthcoming) for details). Under neo-additive preferences the DM's evaluation of the mortality risk, is equal to:[4]

$$W_{\tilde{\varepsilon}} = (1-a)E((1-\tilde{p})U_l(w) + \tilde{p}U_d(w)) + \frac{a-b}{2} max[(1-\tilde{p})U_l(w) + \tilde{p}U_d(w)]$$

$$+ \frac{a+b}{2} min[(1-\tilde{p})U_l(w) + \tilde{p}U_d(w)]. \quad (7)$$

[4] We interpret neo-additive preferences in a setting where the possible values of $\tilde{\varepsilon}$ are the states of the world.

In Eq. (7) $a \in [0,1]$ and $b \in [-a, a]$. A neo-additive DM gives weight $(1-a)$ to the expected utility of a random variable $\tilde{p}$, weight $\frac{a-b}{2}$ to the maximum (expected) utility that he can obtain and weight $\frac{a+b}{2}$ to the minimum (expected) utility that he can obtain. Expected utility is the special case of Eq. (7) with $a = b = 0$. Baillon (forthcoming) shows that his model-free definition of ambiguity aversion, which is equivalent to $\varphi'' < 0$ under the smooth model, is equivalent to $b > 0$ under neo-additive preferences. His definition of ambiguity prudence, which is equivalent to $\varphi''' > 0$ under the smooth model, is equivalent to $a > 0$ under neo-additive preferences.

For a random variable $\tilde{\varepsilon}$ the maximum expected utility is obtained for the lowest mortality risk, i.e. for the lowest value of $\tilde{\varepsilon}$. Denote the absolute value of this by ε_{min}. Similarly, the minimum expected utility is obtained for the highest mortality risk, i.e. for the highest value of $\tilde{\varepsilon}$. Denote the absolute value of this by ε_{max}. As we consider changes in ambiguity perception with mean zero, Eq. (7) can then be written as:

$$(1-a)((1-p)U_l(w) + pU_d(w)) + \frac{a-b}{2}\big[\big(1-(p-\varepsilon_{min})\big)U_l(w) + (p-\varepsilon_{min})U_d(w)\big]$$
$$+ \frac{a+b}{2}\big[\big(1-(p+\varepsilon_{max})\big)U_l(w) + (p+\varepsilon_{max})U_d(w)\big]. \quad (8)$$

Totally differentiating Eq. (8) with respect to p and w gives the VSL under neo-additive preferences:

$$VSL_{\tilde{\varepsilon}_i} = \frac{dw}{dp}$$

$$= \frac{U_l(w) - U_d(w)}{(1-p)U_l'(w) + pU_d'(w) + \left(U_l'(w) + U_d'(w)\right)\left[\frac{a-b}{2}\varepsilon_{min} - \frac{a+b}{2}\varepsilon_{max}\right]} \quad (9)$$

As $a \in [0,1]$ and $b \in [-a, a]$, it follows that if the DM is ambiguity averse $(b > 0)$ the sign of the term $\left[\frac{a-b}{2}\varepsilon_{min} - \frac{a+b}{2}\varepsilon_{max}\right]$ is indeterminate and depends on the relative magnitudes of ε_{min} and ε_{max}. Hence, under neo-additive preferences we cannot replicate Treich's result that an ambiguity averse DM will always have a higher VSL under ambiguity than under no ambiguity. Two points are worth making. First, if $\varepsilon_{min} = \varepsilon_{max}$ then ambiguity aversion implies that $\left[\frac{a-b}{2}\varepsilon_{min} - \frac{a+b}{2}\varepsilon_{max}\right]$ is negative and that ambiguity leads to an increase in the VSL. Second, under Schmeidler's (1989) definition of ambiguity aversion,[5] $a = b$ and an ambiguity averse DM will always have a higher VSL when ambiguity increases.

Let us now consider what happens if ambiguity increases. Consider two random variables $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ with $\tilde{\varepsilon}_2$ a mean-preserving spread of $\tilde{\varepsilon}_1$. We denote the absolute values of the minima of $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ by $\varepsilon_{1,min}$ and $\varepsilon_{2,min}$ and the maximum values by $\varepsilon_{1,max}$ and $\varepsilon_{2,max}$.

Result 7:

[5] Schmeidler (1989) defines ambiguity aversion as a preference for hedging: if the DM is indifferent between two random variables $\tilde{p}$ and $\tilde{q}$ then he prefers their mixture $\lambda\tilde{p} + (1 - \lambda\tilde{q})$, $0 < \lambda < 1$, to each of these variables. Schmeidler's definition implies that the capacity in Choquet expected utility is convex.

Suppose that the DM has neo-additive preferences and is ambiguity averse ($b > 0$). Let $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ be two random variables with $\tilde{\varepsilon}_2$ a mean-preserving spread of $\tilde{\varepsilon}_1$.

i. if $\varepsilon_{2,min} - \varepsilon_{1,min} \leq \varepsilon_{2,max} - \varepsilon_{1,max}$ then $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$.

ii. if $\varepsilon_{2,min} - \varepsilon_{1,min} = e + k > \varepsilon_{2,max} - \varepsilon_{1,max} = e$ then $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$ if $a < \frac{2e+k}{k} b$.

Result 7 shows that the effect of an increase in ambiguity perception depends on the skewness of the change in ambiguity perception and on the DM's ambiguity prudence. If $\varepsilon_{2,min} - \varepsilon_{1,min} \leq \varepsilon_{2,max} - \varepsilon_{1,max}$ then VSL will always increase after an increase in ambiguity perception. If $\varepsilon_{2,min} - \varepsilon_{1,min}$ exceeds $\varepsilon_{2,max} - \varepsilon_{1,max}$ but the DM's is not too ambiguity prudent (a is not too high) then VSL will increase after an increase in ambiguity perception. If ambiguity prudence is strong compared to ambiguity aversion and $\varepsilon_{2,min} - \varepsilon_{1,min}$ exceeds $\varepsilon_{2,max} - \varepsilon_{1,max}$ then it is possible that an increase in ambiguity perception actually leads to a decrease in VSL. Result 7 also shows that the probability that $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$ increases if the DM becomes more ambiguity averse (b increases ceteris paribus).

Treich (2010) studied the special case of Result 7 where $\tilde{\varepsilon}_1 = 0$. By setting $\varepsilon_{2,max} = \varepsilon_{max}, \varepsilon_{2,min} = \varepsilon_{min}, \varepsilon_{1,max} = \varepsilon_{1,min} = 0$, it follows an ambiguity averse DM will always have a higher VSL under ambiguity than under no ambiguity if $a < \frac{\varepsilon_{max}+\varepsilon_{min}}{\varepsilon_{min}-\varepsilon_{max}} b$.[6]

Result 8 states the results for an ambiguity seeking DM.

RESULT 8:

[6] If $\tilde{\varepsilon}$ is symmetric ($\varepsilon_{min} = \varepsilon_{max}$) or $\varepsilon_{min} < \varepsilon_{max}$ then it is of course always true that $VSL_{\tilde{\varepsilon}} > VSL_r$.

Suppose that the DM has neo-additive preferences and is ambiguity seeking ($b < 0$). Let $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ be two random variables with $\tilde{\varepsilon}_2$ a mean-preserving spread of $\tilde{\varepsilon}_1$.

i. if $\varepsilon_{2,min} - \varepsilon_{1,min} \geq \varepsilon_{2,max} - \varepsilon_{1,max}$ then $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$.

ii. if $\varepsilon_{2,min} - \varepsilon_{1,min} = e < \varepsilon_{2,max} - \varepsilon_{1,max} = e + k$ then $VSL_{\tilde{\varepsilon}_2} < VSL_{\tilde{\varepsilon}_1}$ if $a < -\frac{2e+k}{k} b$.

Hence, we observe, as in Result 5, that an ambiguity seeking DM's VSL will decrease when his ambiguity perception increases if he is not too ambiguity prudent.

7.1. Biased beliefs

Thus far we have assumed that ambiguity aversion does not lead to a bias in the DM's beliefs. That is the random variables $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ have expectation zero. In this subsection we will briefly consider the case where ambiguity leads to a bias in the DM's beliefs. Let $E(\tilde{\varepsilon}_1) = g_1$ and $E(\tilde{\varepsilon}_2) = g_2$. If $g_1 \neq g_2$, $\tilde{\varepsilon}_2$ cannot be a mean-preserving spread of $\tilde{\varepsilon}_1$. We define $\tilde{\varepsilon}_2$ as more ambiguous than $\tilde{\varepsilon}_1$ if the interval of possible values that the mortality risk takes under $\tilde{\varepsilon}_1$ is a subset of the interval of possible values under $\tilde{\varepsilon}_2$.[7]

RESULT 9:

Suppose that the DM has neo-additive preferences and is ambiguity averse ($b > 0$). Let $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ be two random variables with the support of $\tilde{\varepsilon}_1$ contained in the support of $\tilde{\varepsilon}_2$.

i. if $\varepsilon_{2,min} - \varepsilon_{1,min} = \varepsilon_{2,max} - \varepsilon_{1,max} = e$ then $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$ if $b > \frac{g_1 - g_2}{e}$.

[7] Results 7 and 8 also hold under this definition of more ambiguous.

ii. if $\varepsilon_{2,min} - \varepsilon_{1,min} = e < \varepsilon_{2,max} - \varepsilon_{1,max} = e + k$ then $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$ if $a > -\frac{(2e+k)b-2(g_1-g_2)}{k}$.

iii. if $\varepsilon_{2,min} - \varepsilon_{1,min} = e + k > \varepsilon_{2,max} - \varepsilon_{1,max} = e$ then $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$ if $a < \frac{(2e+k)b-2(g_1-g_2)}{k}$.

Result 9 shows that if $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ are equally biased then Result 7 still holds. Part (ii) holds because $-\frac{(2e+k)b}{k} < 0$ if the DM is ambiguity averse and $a \geq 0$. If $\tilde{\varepsilon}_2$ is more upward biased than $\tilde{\varepsilon}_1$, i.e., $g_2 > g_1$ then the conditions for the VSL to increase with an increase in ambiguity become less stringent. The intuition is that if $g_2 > g_1$, the DM expects a higher mortality risk under $\tilde{\varepsilon}_2$ and we know from the dead anyway effect that higher (expected) mortality risks increase VSL. However, if $\tilde{\varepsilon}_2$ is less upward biased than $\tilde{\varepsilon}_1$, i.e., $g_2 < g_1$ then the conditions for the VSL to increase with an increase in ambiguity become more stringent because of the dead-anyway effect and even in the symmetric case (i) ambiguity aversion alone is not enough to ensure that the VSL will increase.

Result 10 states the results for an ambiguity seeking DM. It is the immediate counterpart of Result 9.

RESULT 10:

Suppose that the DM has neo-additive preferences and is ambiguity seeking ($b < 0$). Let $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ be two random variables with the support of $\tilde{\varepsilon}_1$ contained in the support of $\tilde{\varepsilon}_2$.

i. if $\varepsilon_{2,min} - \varepsilon_{1,min} = \varepsilon_{2,max} - \varepsilon_{1,max} = e$ then $VSL_{\tilde{\varepsilon}_2} < VSL_{\tilde{\varepsilon}_1}$ if $b < \frac{g_1-g_2}{e}$.

ii. if $\varepsilon_{2,min} - \varepsilon_{1,min} = e < \varepsilon_{2,max} - \varepsilon_{1,max} = e + k$ then $VSL_{\tilde{\varepsilon}_2} < VSL_{\tilde{\varepsilon}_1}$ if $a < -\frac{(2e+k)b-2(g_1-g_2)}{k}$.

iii. if $\varepsilon_{2,min} - \varepsilon_{1,min} = e + k > \varepsilon_{2,max} - \varepsilon_{1,max} = e$ then $VSL_{\tilde{\varepsilon}_2} < VSL_{\tilde{\varepsilon}_1}$ if $a > \frac{(2e+k)b-2(g_1-g_2)}{k}$.

For an ambiguity seeker, if $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ are equally biased then Result 8 still holds. If $\tilde{\varepsilon}_2$ is more upward biased than $\tilde{\varepsilon}_1$ ($g_2 > g_1$) then the conditions for the VSL to decrease with an increase in ambiguity become more stringent. The intuition, again, is that the dead anyway effect goes in the direction of a larger value of $VSL_{\tilde{\varepsilon}_2}$ compared to $VSL_{\tilde{\varepsilon}_1}$. If $\tilde{\varepsilon}_2$ is less upward biased than $\tilde{\varepsilon}_1$ ($g_2 < g_1$) then the conditions for the VSL to increase with an increase in ambiguity become less stringent. Parts (i) and (iii) then always hold and the restriction on prudence in Part (ii) becomes weaker.

8. Conclusion.

The VSL is an important concept in policy evaluation. The properties of VSL have mainly been studied under risk where probabilities are objectively given. However, in most real-world decisions probabilities are at best vaguely known. The empirical literature shows that people are not neutral towards such ambiguity. Treich (2010) derived that an ambiguity averse DM who behaves according to the smooth ambiguity model of KMM (2005) will have a higher VSL under ambiguity than under no ambiguity. We have shown that this cannot be generalized to general increases in ambiguity. An ambiguity averse DM's VSL may actually be lower in more ambiguous decision situations. To sign the effect of increases in ambiguity, information on higher order ambiguity attitudes is required. We have particularly highlighted the important role of ambiguity prudence. Our results confirm Baillon's (forthcoming) conclusion that ambiguity prudence plays a key role in explaining economic behavior.

The implications of our results for cost-benefit analysis depend on whether deviations from ambiguity neutrality are considered normative or not. Cost-benefit analysis is a prescriptive exercise but VSL is estimated by eliciting people's preferences for mortality risk reductions, which is a descriptive task. If ambiguity aversion is viewed as irrational then our results indicate what the bias in the estimated VSL will be. If an increase in perceived ambiguity is skewed to the right, i.e. people perceive more situations with a high risk to human life, then they will react too strongly to the ambiguity leading to estimates of VSL that are too high. Examples may be the reaction to the mad cow disease or the threat of terrorist attacks. If the distribution is skewed to the left, i.e. people perceive more situations in which the risk to human life is lower, and they are not too ambiguity prudent, their estimated VSL will be too low. An example may be climate change. On the other hand, if ambiguity aversion is considered rational (see for example Gilboa and Marinacchi 2013) our results guide policy as to how the VSL used in policy evaluations should be adjusted to changes in ambiguity.

Empirical research on ambiguity prudence is still thin on the ground. Baillon et al. (2016) tested ambiguity prudence and found support for it. However, they only obtained qualitative support for ambiguity prudence and did not quantify its intensity. Our results highlight that such quantification is required to understand the effects of (increases in) ambiguity on VSL. Ebert and Wiesen (2014) showed how the intensity of risk prudence can be measured. Extending their research to ambiguity is a worthwhile topic for future research.

Appendix: Proofs

Proof of Result 2.

Define $\widetilde{Z}_i = (1-\tilde{p}_i)U_l(w) + \tilde{p}_i U_d(w)$ and $\widetilde{Z}_i' = (1-\tilde{p}_i)U_l'(w) + \tilde{p}_i U_d'(w)$, $i = 1,2$. By Eq. (5), $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$ if

$$\frac{E\left[\varphi'\left(\tilde{Z}_2\right)\right]}{E\left[\varphi'\left(\tilde{Z}_1\right)\right]} > \frac{E\left[\tilde{Z}_2'\varphi'\left(\tilde{Z}_2\right)\right]}{E\left[\tilde{Z}_1'\varphi'\left(\tilde{Z}_1\right)\right]}. \qquad (A1)$$

A sufficient condition to obtain Eq. (A1) is

$$\frac{E\left[\varphi'\left(\tilde{Z}_2\right)\right]}{E\left[\varphi'\left(\tilde{Z}_1\right)\right]} > 1 > \frac{E\left[\tilde{Z}_2'\varphi'\left(\tilde{Z}_2\right)\right]}{E\left[\tilde{Z}_1'\varphi'\left(\tilde{Z}_1\right)\right]}. \qquad (A2)$$

Let $Z_0 = (1-p)U_l(w) + pU_d(w)$, $Z_0' = (1-p)U_l'(w) + pU_d'(w)$, $\Delta v = U_l - U_d$, and $\Delta v' = U_l' - U_d'$. Define the following functions: $g(\varepsilon) = \varphi'(Z_0 - \varepsilon\Delta v)$, $z(\varepsilon) = Z_0' - \varepsilon\Delta v'$, and $H(\varepsilon) = g(\varepsilon)z(\varepsilon)$. Eq. (A2) can then be rewritten as:

$$\frac{E[g(\tilde{\varepsilon}_2)]}{E[g(\tilde{\varepsilon}_1)]} > 1 > \frac{E[H(\tilde{\varepsilon}_2)]}{E[H(\tilde{\varepsilon}_1)]}. \qquad (A3)$$

According to Theorem 2 in Rothschild and Stiglitz (1970, p. 237),

$$\frac{E[g(\tilde{\varepsilon}_2)]}{E[g(\tilde{\varepsilon}_1)]} > 1$$

if the second derivative of g is positive. Hence, $g'' = (-\Delta v)^2 \varphi'''(Z_0 - \varepsilon\Delta v) > 0$, which holds if $\varphi'''(x) > 0$ for all x, i.e. if the DM is ambiguity prudent. Theorem 2 in Rothschild and Stiglitz's also implies that

$$1 > \frac{E[H(\tilde{\varepsilon}_2)]}{E[H(\tilde{\varepsilon}_1)]}$$

if the second derivative of H is negative. Differentiating H twice and rearranging terms gives:

$$-\frac{\varphi'''(Z_0 - \varepsilon\Delta v)}{\varphi''(Z_0 - \varepsilon\Delta v)} < 2\frac{\Delta v'}{\Delta v(Z_0' - \varepsilon\Delta v')}. \qquad (A4)$$

Define $S = 2\frac{\Delta v'}{\Delta v(Z_0' - \varepsilon\Delta v')}$ and let S^* be the minimum of $2\frac{\Delta v'}{\Delta v(Z_i' - \varepsilon_i\Delta v')}$ where $Z_i' = (1 - p_i)U_l' + p_i U_d'$ and ε_i varies over the realizations of $\tilde{Z}_i'$ and $\tilde{\varepsilon}_i$. S^* is positive because Δv and $\Delta v'$ are both positive and the second term in the denominator is positive because $0 \leq p_i \leq 1$ for all realizations of $\tilde{p}_i, i = 1,2$. The strictest constraint in (A4) is obtained for the minimum value of S^*. As $\tilde{\varepsilon}_2$ is a mean-preserving transformation of $\tilde{\varepsilon}_1$ this minimum value is obtained for $\varepsilon = \min(\tilde{\varepsilon}_2)$.

If $\varphi''' = 0$, then $H''(\varepsilon) < 0$ and $g''(\varepsilon) = 0$ and, thus,

$$\frac{E[g(\tilde{\varepsilon}_2)]}{E[g(\tilde{\varepsilon}_1)]} = 1 > \frac{E[H(\tilde{\varepsilon}_2)]}{E[H(\tilde{\varepsilon}_1)]},$$

which also implies that $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$.

■

Proof of Result 3.

We first show that mixed ambiguity averse DMs will dislike n^{th} order increases in ambiguity.

LEMMA A1*:* For all changes in ambiguity perception $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ and for all n: $\tilde{\varepsilon}_2 \succcurlyeq_n \tilde{\varepsilon}_1$ iff every mixed ambiguity averse DM prefers $\tilde{\varepsilon}_1$ to $\tilde{\varepsilon}_2$.

Proof. The proof is very similar to Ekern's (1980) derivation on p. 331. First note that even though φ is defined over expected utility in the smooth model, in our setup with only two utilities U_l and U_d there is a one-to-one negative relationship between p and expected utility. Suppose first that $\tilde{\varepsilon}_2 \succcurlyeq_n \tilde{\varepsilon}_1$. The DM will prefer $\tilde{\varepsilon}_1$ to $\tilde{\varepsilon}_2$ if $\int_a^b \varphi(p)dF_2(p) - \int_a^b \varphi(p)dF_1(p) > 0$. Repeated application of integration by parts gives:

$$\int_a^b \varphi(p)dF_2(p) - \int_a^b \varphi(p)dF_1(p) = \varphi(b)\big(F_2(b) - F_1(b)\big)$$
$$+ \sum_{k=2}^{n} (-1)^{k-1}\varphi^{(k-1)}\left(F_2^k(b) - F_1^k(b)\right)$$
$$+ \int_a^b (-1)^n \varphi^{(n)}\big(F_2^n(p) - F_1^n(p)\big)\,dp \qquad (A5).$$

Suppose the DM is mixed ambiguity averse. Then it follows immediately from Definition 1 that $\int_a^b \varphi(p)dF_2(p) - \int_a^b \varphi(p)dF_1(p) > 0$ and thus that $\tilde{\varepsilon}_1 \succcurlyeq \tilde{\varepsilon}_2$ where $\succcurlyeq$ denotes the DM's preference relation.

For the reverse implication, suppose that every mixed ambiguity averter prefers $\tilde{\varepsilon}_1$to $\tilde{\varepsilon}_2$, but that not $\tilde{\varepsilon}_2 \succcurlyeq_n \tilde{\varepsilon}_1$. Then there must be some $m < n$ such that $F_2^m(b) - F_1^m(b) < 0$. Then mixed ambiguity averse DMs for whom $-\frac{\varphi^n}{\varphi^{(m-1)}}$ converges to zero will prefer $\tilde{\varepsilon}_2$ to $\tilde{\varepsilon}_1$, a contradiction. *Q.E.D.*

Define the functions g, z, and H as in the proof of Result 2. It follows that for every $n \geq 2$ $g^{(n)} = (\Delta v)^n \varphi^{(n+1)}(Z_0 - \varepsilon\Delta v)$, $z^{(n)} = 0$, and $H^{(n)} = nz'(\varepsilon)g^{(n-1)}(\varepsilon) + z(\varepsilon)g^{(n)}(\varepsilon) = n(\Delta v')g^{(n-1)}(\varepsilon) + z(\varepsilon)g^{(n)}(\varepsilon)$.

It is still true that Eq. (A3) is a sufficient condition to get $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_2}$. Equation (A3) holds if $E[g(\tilde{\varepsilon}_2)] > E[g(\tilde{\varepsilon}_1)]$ and $E[H(\tilde{\varepsilon}_2)] < E[H(\tilde{\varepsilon}_1)]$. Because g is a function for which $g^{(n)} > 0$ if n is even and $g^{(n)} < 0$ if n is odd, it follows from Eq. (A5) that $E[g(\tilde{\varepsilon}_2)] > E[g(\tilde{\varepsilon}_1)]$. From Lemma A1, $E[H(\tilde{\varepsilon}_2)] < E[H(\tilde{\varepsilon}_1)]$ will hold if H is a function that has $H^{(n)} > 0$ if n is odd and $H^{(n)} < 0$ if n is even.

Now, $H^{(n)} = n(\Delta v')g^{(n-1)}(\varepsilon) + z(\varepsilon)g^{(n)}(\varepsilon) = n(\Delta v')(\Delta v)^{n-1}\varphi^{(n)}(Z_0 - \varepsilon\Delta v) + (Z_0' - \varepsilon\Delta v')(\Delta v)^n \varphi^{(n+1)}(Z_0 - \varepsilon\Delta v)$. If n is odd this expression should be positive. That is, $(Z_0' - \varepsilon\Delta v')(\Delta v)^n \varphi^{(n+1)}(Z_0 - \varepsilon\Delta v) > -n(\Delta v')(\Delta v)^{n-1}\varphi^{(n)}(Z_0 - \varepsilon\Delta v)$. As $\varphi^{(n)}$ and Δv are both positive, it follows that $-\frac{\varphi^{(n+1)}}{\varphi^{(n)}} < \frac{n(\Delta v')}{(\Delta v)(Z_0' - \varepsilon\Delta v')} = nS^*$.

If n is even $H^{(n)}$ should be negative. That is, $(Z_0' - \varepsilon\Delta v')(\Delta v)^n \varphi^{(n+1)}(Z_0 - \varepsilon\Delta v) < -n(\Delta v')(\Delta v)^{n-1}\varphi^{(n)}(Z_0 - \varepsilon\Delta v)$. As $\varphi^{(n)}$ is negative and Δv is positive, it follows that $-\frac{\varphi^{(n+1)}}{\varphi^{(n)}} < \frac{n(\Delta v')}{(\Delta v)(Z_0' - \varepsilon\Delta v')} = nS^*$. ■

Proof of Result 4.

$VSL_{\tilde{\varepsilon}_2} < VSL_{\tilde{\varepsilon}_1}$ is equivalent to

$$\frac{E[\varphi'(\tilde{Z}_2)]}{E[\varphi'(\tilde{Z}_1)]} < \frac{E[\tilde{Z}_2'\varphi'(\tilde{Z}_2)]}{E[\tilde{Z}_1'\varphi'(\tilde{Z}_1)]}. \qquad (A6)$$

Because $\tilde{\varepsilon}_1$ is degenerate ($\tilde{\varepsilon}_1 = 0$), $\tilde{Z}_1'$ is constant. Then the denominator of Eq. (A6) becomes $E[\tilde{Z}_1']E[\varphi'(\tilde{Z}_1)]$. Because $\tilde{\varepsilon}_2$ is a mean-preserving spread of $\tilde{\varepsilon}_1$, $E[\tilde{Z}_1'] = E[\tilde{Z}_2']$. Substituting $E[\tilde{Z}_1'\varphi'(\tilde{Z}_1)] = E[\tilde{Z}_1']E[\varphi'(\tilde{Z}_1)]$ and $E[\tilde{Z}_1'] = E[\tilde{Z}_2']$ into Eq. (A6) gives $VSL_{\tilde{\varepsilon}_2} < VSL_{\tilde{\varepsilon}_1}$ iff $E[\tilde{Z}_2']E[\varphi'(\tilde{Z}_2)] < E[\tilde{Z}_2'\varphi'(\tilde{Z}_2)]$. The last inequality is holds if the covariance between $\tilde{Z}_2'$ and $\varphi'(\tilde{Z}_2)$ is positive, which holds because the DM is ambiguity seeking. ■

Proof of Result 5.

The proof is very similar to that of Result 2. Define the functions, $g(\varepsilon)$, $z(\varepsilon)$, and $H(\varepsilon)$ as in the proof of Result 2. By Eq. (5), a sufficient condition to have, $VSL_{\tilde{\varepsilon}_2} < VSL_{\tilde{\varepsilon}_1}$ is

$$\frac{E[g(\tilde{\varepsilon}_2)]}{E[g(\tilde{\varepsilon}_1)]} < 1 < \frac{E[H(\tilde{\varepsilon}_2)]}{E[H(\tilde{\varepsilon}_1)]}. \qquad (A7)$$

Equation (A7) holds when $g(\varepsilon)$ has a negative second derivative and $H(\varepsilon)$ has a positive second derivative. $g''(\varepsilon) = (\Delta v)^2 \varphi'''(Z_0 - \varepsilon\Delta v) < 0$, which holds if $\varphi'''(x) < 0$ for all x, i.e. if the DM is ambiguity imprudent.

$H''(\varepsilon) = (\Delta v)\varphi'''(Z_0 - \varepsilon\Delta v)(Z_0' - \varepsilon\Delta v') + 2(\Delta v')\varphi'''(Z_0 - \varepsilon\Delta v)$, which is positive if Eq. (A4) holds.

If $\varphi''' = 0$, then $H''(\varepsilon) > 0$ and $g''(\varepsilon) = 0$ and, thus,

$$\frac{E[g(\tilde{\varepsilon}_2)]}{E[g(\tilde{\varepsilon}_1)]} = 1 < \frac{E[H(\tilde{\varepsilon}_2)]}{E[H(\tilde{\varepsilon}_1)]},$$

which also implies that $VSL_{\tilde{\varepsilon}_2} < VSL_{\tilde{\varepsilon}_1}$. ■

Proof of Result 6.

The proof is identical to the proof of Result 3.

Proof of Result 7.

It is immediate from Eq. (9) that the comparison between $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ depends on the relative magnitudes of $\left[\frac{a-b}{2}\varepsilon_{1,min} - \frac{a+b}{2}\varepsilon_{1,max}\right]$ and $\left[\frac{a-b}{2}\varepsilon_{2,min} - \frac{a+b}{2}\varepsilon_{2,max}\right]$. We have $VSL_{\tilde{\varepsilon}_2} \gtreqless VSL_{\tilde{\varepsilon}_1}$ iff $\left[\frac{a-b}{2}\varepsilon_{2,min} - \frac{a+b}{2}\varepsilon_{2,max}\right] \lesseqgtr \left[\frac{a-b}{2}\varepsilon_{1,min} - \frac{a+b}{2}\varepsilon_{1,max}\right]$ or, equivalently, iff $\left[\frac{a-b}{2}\left(\left(\varepsilon_{2,min} - \varepsilon_{1,min}\right)\right)\right] - \left[\frac{a+b}{2}\left(\left(\varepsilon_{2,max} - \varepsilon_{1,max}\right)\right)\right] \lesseqgtr 0$.

Part (i). Suppose $\varepsilon_{2,min} - \varepsilon_{1,min} = \varepsilon_{2,max} - \varepsilon_{1,max} = e$. Then $\left[\frac{a-b}{2}\left(\left(\varepsilon_{2,min} - \varepsilon_{1,min}\right)\right)\right] - \left[\frac{a+b}{2}\left(\left(\varepsilon_{2,max} - \varepsilon_{1,max}\right)\right)\right] = -be < 0$. Hence, $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$.

Suppose $\varepsilon_{2,min} - \varepsilon_{1,min} = e, \varepsilon_{2,max} - \varepsilon_{1,max} = e + k, k > 0$. Then $\left[\frac{a-b}{2}\left(\left(\varepsilon_{2,min} - \varepsilon_{1,min}\right)\right)\right] - \left[\frac{a+b}{2}\left(\left(\varepsilon_{2,max} - \varepsilon_{1,max}\right)\right)\right] = -be - k\frac{a+b}{2} < 0$. Hence, $VSL_{\tilde{\varepsilon}_2} > VSL_{\tilde{\varepsilon}_1}$.

Part (ii). Denote $\varepsilon_{2,min} - \varepsilon_{1,min} = e + k, \varepsilon_{2,max} - \varepsilon_{1,max} = e, k > 0$. Then $\left[\frac{a-b}{2}\left(\left(\varepsilon_{2,min} - \varepsilon_{1,min}\right)\right)\right] - \left[\frac{a+b}{2}\left(\left(\varepsilon_{2,max} - \varepsilon_{1,max}\right)\right)\right] = -b\varepsilon + k\,\frac{a-b}{2}$. This expression is negative if $a < \frac{2b\varepsilon}{k} + b$. $VSL_{\tilde{\varepsilon}} \gtreqless VSL_r$ iff $a \lesseqgtr \frac{2e+k}{k} b$. ■

Proof of Result 8.

This follows straightforwardly from the proof of Result 7 by setting $b < 0$.

Proof of Result 9

Eq. (9) shows that we have $VSL_{\tilde{\varepsilon}_2} \gtreqless VSL_{\tilde{\varepsilon}_1}$ iff $(g_1 - g_2) + \left[\frac{a-b}{2}\left(\left(\varepsilon_{2,min} - \varepsilon_{1,min}\right)\right)\right] - \left[\frac{a+b}{2}\left(\left(\varepsilon_{2,max} - \varepsilon_{1,max}\right)\right)\right] \lesseqgtr 0$.

Part (i). Suppose $\varepsilon_{2,min} - \varepsilon_{1,min} = \varepsilon_{2,max} - \varepsilon_{1,max} = e$. Then $(g_1 - g_2) + \left[\frac{a-b}{2}\left(\left(\varepsilon_{2,min} - \varepsilon_{1,min}\right)\right)\right] - \left[\frac{a+b}{2}\left(\left(\varepsilon_{2,max} - \varepsilon_{1,max}\right)\right)\right] = (g_1 - g_2) - be$. Hence, $VSL_{\tilde{\varepsilon}_2} \gtreqless VSL_{\tilde{\varepsilon}_1}$ iff $b \gtreqless \frac{g_1 - g_2}{e}$.

Part (ii). Suppose $\varepsilon_{2,min} - \varepsilon_{1,min} = e < \varepsilon_{2,max} - \varepsilon_{1,max} = e + k$. $k > 0$. Then $(g_1 - g_2) + \left[\frac{a-b}{2}\left(\left(\varepsilon_{2,min} - \varepsilon_{1,min}\right)\right)\right] - \left[\frac{a+b}{2}\left(\left(\varepsilon_{2,max} - \varepsilon_{1,max}\right)\right)\right] = (g_1 - g_2) - be - k\,\frac{a+b}{2}$. Hence, $VSL_{\tilde{\varepsilon}_2} \gtreqless VSL_{\tilde{\varepsilon}_1}$ iff $a \gtreqless -\frac{(2e+k)b - 2(g_1 - g_2)}{k}$.

Part (iii). Suppose $\varepsilon_{2,min} - \varepsilon_{1,min} = e + k > \varepsilon_{2,max} - \varepsilon_{1,max} = e$. $k > 0$. Then $(g_1 - g_2) + \left[\frac{a-b}{2}\left(\left(\varepsilon_{2,min} - \varepsilon_{1,min}\right)\right)\right] - \left[\frac{a+b}{2}\left(\left(\varepsilon_{2,max} - \varepsilon_{1,max}\right)\right)\right] = (g_1 - g_2) - be + k\,\frac{a-b}{2}$. Hence, $VSL_{\tilde{\varepsilon}_2} \gtreqless VSL_{\tilde{\varepsilon}_1}$ iff $a \lesseqgtr \frac{(2e+k)b - 2(g_1 - g_2)}{k}$.

Proof of Result 10.

This follows straightforwardly from the proof of Result 9 by setting $b < 0$.

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