Quantitative Models in Marketing Research: Corrections and Additions*

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Chapter 2

- Page 22, 11th line from above: Replace "10–20 guilders" by "0–20 guilders".
- Page 25, 14th line from above: Replace "25–50 days" by "0–50 days".
- Page 26, Table 2.6, Notes a,b,c: Replace "the brand" by "liquid detergent".

Chapter 3

- Page 32: Replace the two sentences before equation (3.8) by
  A parameter β_k measures the effect of a variable $x_{k,t}$ on Y_t, $k \in \{1, \ldots, K\}$ assuming that this variable is uncorrelated with the other explanatory variables and ε_t. This can be seen from the partial derivative
- Page 32: Replace sentence after equation (3.8) by
  Note that if another variable $x_{l,t}$ is a function of $x_{k,t}$, for example, $x_{l,t} = x_{k,t}^2$, this partial effect will also depend on the partial derivative of y_t to $x_{l,t}$ and the corresponding β_l parameter.

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- Page 34, equation (3.13): The correct equation is

$$\frac{\partial \sum_{t=1}^{T}(y_t - X_t\beta)^2}{\partial\beta} = -\mathbf{2}\sum_{t=1}^{T} X_t'(y_t - X_t\beta) = 0.$$

- Page 37, equation (3.24): Divide $\hat{\mathcal{I}}$ by T. The correct equation is

$$\sqrt{T}(\hat{\theta} - \theta) \overset{a}{\sim} \mathrm{N}(0, (\hat{\mathcal{I}}/T)^{-1}).$$

- Page 38, first line of equation (3.28). The correct equation is

$$\frac{\partial l(\beta, \sigma^2)}{\partial\beta} = -\mathbf{1}\sum_{t=1}^{T} \frac{1}{\sigma^2} X_t'(y_t - X_t\beta) = 0.$$

- Page 40, equation (3.40): It is better to also add the cross-terms to the test equation and hence the test regression becomes

$$\hat{\varepsilon}_t = \nu_0 + \sum_{k=1}^{K} \nu_k x_{i,k} + \sum_{k=1}^{K}\sum_{j=k}^{K} \gamma_{kj} x_{i,k} x_{i,j} + w_t$$

  The actual test statistic is in this case the joint F-test for the significance of final $1/2K(K+1)$ variables in (3.40).

- Page 42, equation (3.43). The correct equation is

$$R^2 = 1 - \frac{\sum_{t=1}^{T} \hat{\varepsilon}_t^2}{\sum_{t=1}^{T}(y_t - \bar{y})^2},$$

  where $\bar{y}$ denotes the average value of y_t.

- Page 42, equation (3.44): Remove $1/T$ and hence the formula becomes

$$\mathrm{AIC} = -2l(\hat{\theta}) + 2n$$

- Page 42, equation (3.45): Remove $1/T$ and hence the formula becomes

$$\mathrm{BIC} = -2l(\hat{\theta}) + n \log T$$

- Page 48, equation (3.62): Replace $\sum_{k=1}^{K}(\beta_{k,j} - \beta_{K,J}) \log x_{k,j,t}$ by $\sum_{k=1}^{K}(\beta_{k,j} \log x_{k,j,t} - \beta_{K,J} \log x_{K,J,t})$.

- Page 48, 8th line from below: Delete sentence "Also, for similar reasons, one of the $\beta_{k,j}$ parameters is not identified for each k.

- Page 48, 6th line from below: Remove "and $\beta^*_{k,j} = \beta_{k,j} - \beta_{k,J}$"

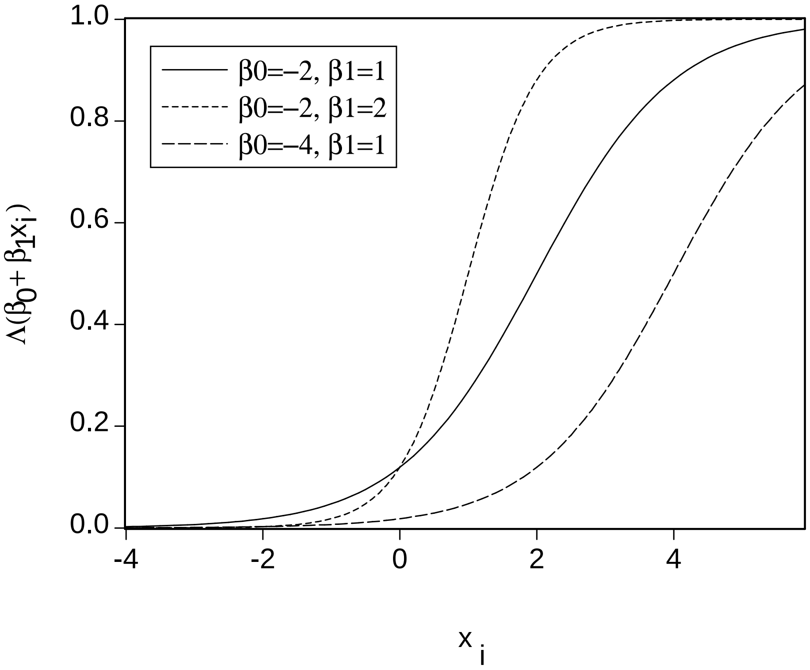


Figure 4.3 with missing information

Chapter 4

- Page 53, equation (4.11): Replace $\varepsilon_{A,i} - \varepsilon_{B,i}$ by $\varepsilon_{B,i} - \varepsilon_{A,i}$.
- Page 54, equation (4.12): Add the following sentence after the equation: where we use in the final line of the equation the assumption that the distribution of ε_i is symmetric.
- Page 56, equation (4.17): Left parenthesis is missing in the final line of the equation. The correct equation (4.17) is:

$$= \frac{\exp\Big(\beta_1(\frac{\beta_0}{\beta_1} + x_i)\Big)}{1 + \exp\Big(\beta_1(\frac{\beta_0}{\beta_1} + x_i)\Big)}$$

- Page 56, Figure 4.3 (paperback version only): The β parameters are missing. The correct picture is provided.
- Page 60, equation (4.40): This equation shows minus the expected value of the Hessian matrix (information matrix) instead of the Hessian matrix. The correct text has to be: ... and the expected value of the Hessian is given by

$$\mathrm{E}[H(\beta)] = \mathrm{E}\Big[\frac{\partial^2 l(\beta)}{\partial\beta\partial\beta'}\Big] = -\sum_{i=1}^{N} \frac{\phi(X_i\beta)^2}{\Phi(X_i\beta)(1 - \Phi(X_i\beta))} X_i' X_i.$$

The asymptotic covariance matrix of the parameters β can be estimated by $(-\mathrm{E}[H(\beta)])^{-1}$, evaluated in the ML estimate $\hat{\beta}$.

- Page 63, below equation (4.49): Simulations in Davidson & MacKinnon (1984, *Journal of Econometrics*) show that the best statistic to use in finite samples is the explained sum of squares from regression (4.49). This test statistic is also used in the application in Section 4.4.

- Page 64, 1st line below equation (4.53): Replace "Notice that the lower bound ... equal to zero." by "The lower bound value of this R^2 is equal to 0. The upper bound is however only equal to 1 when $\Pr[Y_i = y_i|X_i] = 1$ for all i which will hardly happen in practical situations.".

- Page 64, equation (4.55): Remove $1/N$ and hence the formula becomes

$$\mathrm{AIC} = -2l(\hat{\theta}) + 2n$$

- Page 65, equation (4.56): Remove $1/N$ and hence the formula becomes

$$\mathrm{BIC} = -2l(\hat{\theta}) + n \log N$$

- Page 72, 4th paragraph, 1st sentence: Replace "drawings" by "draws".

- Page 74, 2nd line below equation (4.74): Remove "log". The correct sentence is: For the Logit model, one can easily find the link between $\Pr_s[Y_i = 1]$ and $\Pr_p[Y_i = 1]$ because this model holds that the odds ratio is ...

Chapter 5

- Page 78: Replace "nominator" by "numerator".

- Page 80, equation (5.9): The second x_i variable in the numerator has to be inside parentheses. The correct equation is:

$$\begin{aligned}\frac{\partial \Pr[Y_i = j|X_i]}{\partial x_i} &= \frac{(1 + \sum_{l=1}^{J-1} \exp(\beta_{0,l} + \beta_{1,l} x_i)) \exp(\beta_{0,j} + \beta_{1,j} \mathbf{x_i}) \beta_{1,j}}{(1 + \sum_{l=1}^{J-1} \exp(\beta_{0,l} + \beta_{1,l} x_i))^2} \\ &\quad - \frac{\exp(\beta_{0,j} + \beta_{1,j} x_i) \sum_{l=1}^{J-1} \exp(\beta_{0,l} + \beta_{1,l} x_i) \beta_{1,l}}{(1 + \sum_{l=1}^{J-1} \exp(\beta_{0,l} + \beta_{1,l} x_i))^2}\end{aligned}$$

- Page 84, equation (5.25): Remove

$$\text{and } \sum_{l=1}^{J} \frac{\partial \Pr[Y_i = j|w_i]}{\partial w_{i,l}} w_{i,l} = 0.$$

This equality is not true.

- Page 85 2nd line below (5.26): Replace the sentence: "Note that it is not possible to modify α into α_j because the z_j variables are in fact already proportional to the choice-specific intercept terms." by "Note that it is not possible to identify the α parameters as restricting $\alpha = 0$ and redefining $\beta_{0,j}$ as $\beta_{0,j} + \alpha z_j - \alpha_J z_j$ for all j provides the same general logit specification. This can also be seen from the odds ratio in (5.27).

- Page 87, equation (5.29): The term $du_{i,j}$ has to be at the end. The correct equation is
$$\int_{-\infty}^{\infty} \int_{-\infty}^{u_{i,j}} \cdots \int_{-\infty}^{u_{i,j}} f(u_{i,1}, \ldots, u_{i,J})\, du_{i,1} \ldots du_{i,j-1}\, du_{i,j+1} \ldots du_{i,J}\, du_{i,j}$$

- Page 87, equation (5.30): The term $\exp(-\varepsilon_{i,j})$ is missing. The correct equation is
$$f(\varepsilon_{i,j}) = \exp(-\varepsilon_{i,j}) \exp(-\exp(-\varepsilon_{i,j})), \text{ for } j = 1, \ldots, J,$$

- Page 99, 3rd line below equation (5.64): Replace "because $\ell(\hat{\theta})$ will never be 0." by "because $\ell(\hat{\theta})$ will never be 0 in practical situations."

- Page 102, 19th line from above: Replace 32.98 by 45.51.

- Page 104, 11th line from above: Replace "0.025 and 0.050" by "–0.46 and –0.40".

- Page 104, 6th line from below: Replace "–0.46 and -0.40" by "0.012 and 0.055".

- Page 110, 7-8th line from above: Correct notation is
```
' Specify log-likelihood for Multinomial Logit model
logl mnl
```

- Page 110, 14th line from above: `exp(xb4)` has to be `exp(xb1)`. The correct line is: `mnl.append denom=1+exp(xb1)+exp(xb2)+exp(xb3)`

Chapter 6

- Page 121, 2nd equation of (6.30): The correct equation is
$$\frac{\partial^2 \ell(\theta)}{\partial \mu_s \partial \mu_l} = \sum_{j=1}^{J-1} \left(\sum_{r=1}^{J-1} \frac{\partial^2 \ell(\theta)}{\partial \alpha_j \alpha_r} \frac{\partial \alpha_j}{\partial \mu_s} \frac{\partial \alpha_r}{\partial \mu_l} + \frac{\partial \ell(\theta)}{\partial \alpha_j} \frac{\partial^2 \alpha_j}{\partial \mu_s \partial \mu_l} \right) \quad \text{for } s, l = 1, \ldots, J-1.$$

- Page 121, equation (6.32): The correct equation is
$$\frac{\partial^2 \alpha_j}{\partial \mu_s \partial \mu_l} = \begin{cases} 2 & \text{if } 1 < s = l \leq j \\ 0 & \text{otherwise.} \end{cases}$$

- Page 121, equation (6.29): To guarantee strict inequalities it is better to consider the transformation

$$\begin{aligned}
\alpha_1 &= \mu_1 \\
\alpha_2 &= \mu_1 + \exp(\mu_2) = \alpha_1 + \exp(\mu_2) \\
\alpha_3 &= \mu_1 + \exp(\mu_2) + \exp(\mu_3) = \alpha_2 + \exp(\mu_3) \\
\vdots \quad & \qquad \vdots \qquad\qquad \vdots \\
\alpha_{J-1} &= \mu_1 + \sum_{j=2}^{J-1} \exp(\mu_j) = \alpha_{J-2} + \exp(\mu_{J-1})
\end{aligned}$$

in which case (6.31) and (6.32) become

$$\frac{\partial \alpha_j}{\partial \mu_s} = \begin{cases} 1 & \text{if } s = 1 \\ \exp(\mu_s) & \text{if } 1 < s \leq j \\ 0 & \text{if } s > j \end{cases}$$

and

$$\frac{\partial \alpha_j}{\partial \mu_s \partial \mu_l} = \begin{cases} \exp(\mu_s) & \text{if } 1 < s = l \leq j \\ 0 & \text{otherwise,} \end{cases}$$

respectively.

- Page 124, equation (6.45): The correct equation is

$$R^2 = \frac{\sum_{i=1}^{N} (\hat{y}_i^* - \bar{y}^*)^2}{\sum_{i=1}^{N} (\hat{y}_i^* - \bar{y}^*)^2 + N\sigma^2},$$

where $\bar{y}^*$ denotes the average value of $\hat{y}_i^*$.

- Page 130: Replace equation (6.48) by

$$\frac{\Pr[Y_i = j+1|X_i]}{\Pr[Y_i = j|X_i]} = \exp(\beta_{0,j+1} + \beta_1 x_i)$$

for $j = 2, \ldots, J$. So if $\beta_1 > 0$ an increase in x_i means that individual i is more likely to choice $j+1$ versus j for $j = 2, \ldots, J-1$. The corresponding choice probabilities are given by

$$\begin{aligned}
\Pr[Y_i = 1|X_i] &= \frac{1}{1 + \sum_{l=2}^{J} \exp(\beta_{0,l}^* + (l-1)\beta_1 x_i)} \\
\Pr[Y_i = j|X_i] &= \frac{\exp(\beta_{0,j}^* + (j-1)\beta_1 x_i)}{1 + \sum_{l=2}^{J} \exp(\beta_{0,l}^* + (l-1)\beta_1 x_i)} \text{ for } j = 2, \ldots, J,
\end{aligned}$$

where $\beta_{0,j}^* = \sum_{l=2}^{j} \beta_{0,l}$. This model a restricted MNL model with base category 1 and where $\beta_{1,j} = (j-1)\beta_1$.

- Page 130: Replace equation (6.49) and the text till "outcome categories" by

$$\Pr[Y_i = j|X_i] = \frac{\exp(\beta_{0,j} + \phi_j(\beta_1 x_{1,i} + \beta_2 x_{2,i}))}{\sum_{l=1}^{J} \exp(\beta_{0,l} + \phi_l(\beta_1 x_{1,i} + \beta_2 x_{2,i}))}$$

  for $j = 1, \ldots, J$, where $\beta_{0,1} = 0$, $\phi_1 = 0$ and $\phi_J = 1$ for identification. The corresponding odds ratios are

$$\frac{\Pr[Y_i = j|X_i]}{\Pr[Y_i = l|X_i]} = \exp(\beta_{0,j} - \beta_{0,l} + (\phi_j - \phi_l)(\beta_1 x_{1,i} + \beta_2 x_{2,i})).$$

  which is again a restricted MNL model. The parameter restriction

$$0 = \phi_1 < \phi_2 < \ldots < \phi_{J-1} < \phi_J = 1$$

  implies a larger effect of the x_i variables for larger values of the categories. If $\phi_j = \phi_l$ the x_i variables cannot distinguish the jth and lth category. The Adjacent Categories Model is a special case.

Chapter 7

- Page 17, 6th line from above: Replace $c + \beta_0 + \beta_1 x_i$ by $-c + \beta_0 + \beta_1 x_1$

- Page 140, 8th line from below: Replace "donates" by "does not donate". The correct sentence is: The probability that an individual does not donate to charity is now given by ...

- Page 143, 7th line from below: Replace $\hat{\beta} = \hat{\gamma}\hat{\xi}$ by $\hat{\beta} = \hat{\gamma}/\hat{\xi}$.

- Page 145, equation (7.33): Replace $I[y_i = 1]$ by $I[y_i > 0]$.

- Page 146, equation (7.35): Replace $\Phi(-X_i\alpha)$ by $\log(\Phi(-X_i\alpha))$. Likewise, replace $(1 - \Phi(-(X_i\alpha + \sigma_{12}\sigma_2^{-2}(y_i - X_i\beta))/\tilde{\sigma})$ by $\log(1 - \Phi(-(X_i\alpha + \sigma_{12}\sigma_2^{-2}(y_i - X_i\beta))/\tilde{\sigma})$.

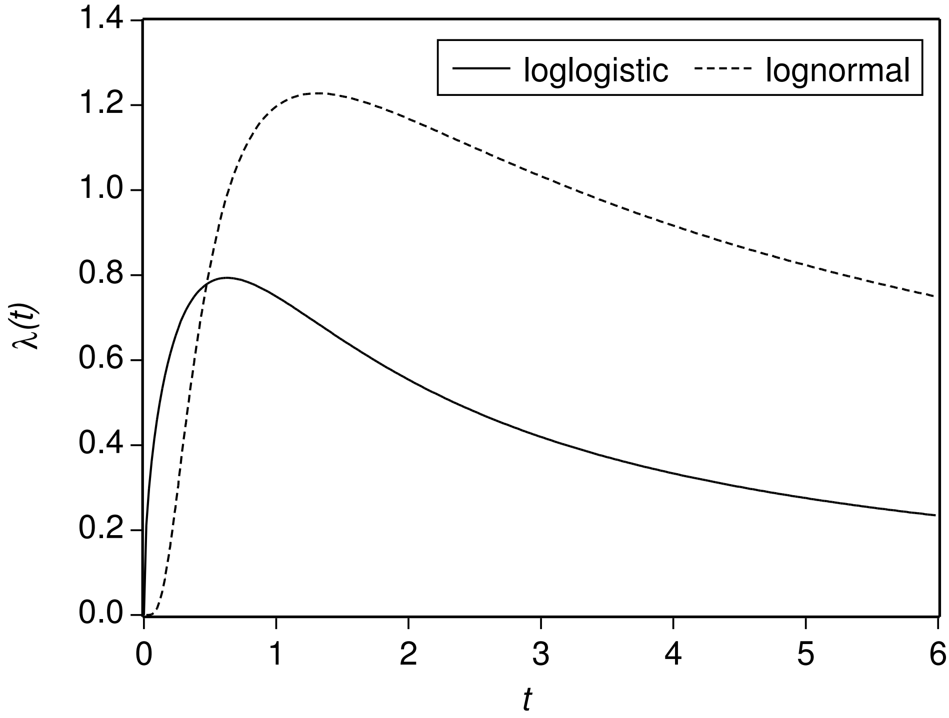
- Page 146, equation (7.35): Replace $I[y_i = 1]$ by $I[y_i > 0]$ (2 times).

- Page 149, equation (7.50): The correct equation is

$$R^2 = \frac{\sum_{i=1}^{N}(\hat{y}_i^* - \bar{y}^*)^2}{\sum_{i=1}^{N}(\hat{y}_i^* - \bar{y}^*)^2 + N\hat{\sigma}^2},$$

  where $\bar{y}^*$ denotes the average value of $\hat{y}_i^*$, ...

- Page 154, 19th line from below: Replace 14.36 by 14.61.



Corrected Figure 8.2

Chapter 8

- Page 159, 7th line from below: Replace $[0, 1]$ by $(0, 1)$. The correct sentence is: ... maps the explanatory variable x_i on the unit interval $(0, 1)$ (see also Section 4.1).
- Page 163, Figure 8.2. The correct graph is provided.
- Page 167, 4th line from above: Replace $\gamma = 0$ with $\gamma = 1$.
- Page 169: Replace $S(t_i|X_i)$ by $S(z_i|X_i)$ in equation (8.36).
- Page 169, equation (8.37): The correction equation is

$$f(z_i|X_i) = \exp(z_i - \exp(z_i)),$$

- Page 169, equation (8.38): The correct equation is

$$l(\theta) = \sum_{i=1}^{N} (d_i f(z_i|X_i) + (1 - d_i) \log S(z_i|X_i)) = \sum_{i=1}^{N} (d_i z_i - \exp(z_i))$$

- Page 170, equation (8.39): The second equation has to be

$$\frac{\partial l(\theta)}{\partial \alpha} = \sum_{i=1}^{N} \frac{d_i z_i - \exp(z_i) z_i}{\alpha}$$

- Page 171, equation (8.46):

$$\frac{\partial \lambda_0(t_i)}{\partial \alpha} = (1 + \alpha \log(t)) t^{\alpha - 1}.$$

- Page 171, equation (8.47): Replace a by α. The correct equation is given by

$$\frac{\partial \Lambda_0(t_i)}{\partial \alpha} = t^\alpha \log(t) \qquad \frac{\partial^2 \Lambda_0(t_i)}{\partial \alpha^2} = t^\alpha (\log(t))^2.$$

- Page 172, equation (8.50): The third line of this equation has to be

$$1 - S(\Lambda^{-1}(E|X_i))$$

- Page 175, 13th line from above: Replace a by α in the expectation. The correct sentence is: For the Proportional Hazard specification, the expectation equals $\exp(-X_i\beta)^{1/\alpha}\Gamma(1 + 1/\alpha)$.

- Page 175, 2nd line of equation (8.61): Replace T by T_i in the denominator. The correct formulation is:

$$= 1 - \frac{\Pr[T_i > t + \Delta t | X_i]}{\Pr[T_i > t | X_i]}$$

- Page 176, Table 8.2: Replace max. log-likelihood value by -11246.24

- Page 177, Table 8.3: Estimate of scale parameter γ has to be 0.019 instead of -0.019.

- Page 178, 3rd line from below: Replace longer by shorter.

- Page 179, equation (8.66): Delete the log operator in the numerator of the second equation. The correct formulation of (8.66) is

$$f(t_i|X_i, v_i) = -\frac{dS(t_i|X_i, v_i)}{dt_i}$$

- Page 183, 3rd line from above: Replace

  ```
  llal.append loglal = censdum*(z+log(a(1)))-exp(z)
  ```

  by

  ```
  llal.append loglal = censdum*(z+log(a(1))-log(interpurch))-exp(z)
  ```

  This mistake does not lead to different estimates but only leads to a different optimal value of the log-likelihood function.

- Page 183, 3rd line of Section 8.A.2: Add `coef(8) b = 1`.

Appendix

- Table A.3, line *Normal distribution*. In the column pdf the term σ^{-1} is missing. It should read

$$\frac{1}{\sigma}\frac{1}{\sqrt{2\pi}}e^{-\frac{(y-\mu)^2}{2\sigma^2}}$$

- Notes of Table A.3, 2nd line from below: $1/\sigma((y-\mu)/\sigma)$ has to be $1/\sigma\phi((y-\mu)/\sigma)$.

Bibliography

- Page 196: correct pages for Amemiya, T. (1981) are 1483–1536.
- Page 198: correct volume number and pages of the Hausman, J.A. and D. Wise (1978) reference are *Econometrica*, **46**, 403–426.

Subject Index

- Page 204: elasticity, Linear Regression model, 32–33
- Page 206: White standard errors, 146